Available online at www.sciencedirect.com

ScienceDirect

Fuzzy Sets and Systems 298 (2016) 109–127

FUZZY sets and systems

www.elsevier.com/locate/fss

CP- and OCF-networks – a comparison

Christian Eichhorn [a,*], Matthias Fey [b,**], Gabriele Kern-Isberner [a,**]

[a] *Chair 1 Computer Science, Technische Universität Dortmund, Dortmund, Germany*
[b] *Technische Universitä t Dortmund, Dortmund, Germany*

Received 11 October 2014; received in revised form 1 April 2016; accepted 11 April 2016

Available online 19 April 2016

Abstract

Network approaches are used to structure, partition and display formalisms in the area of knowledge representation as well as decision making. Known approaches are, for instance, OCF-networks, Bayesian style networks where every variable is annotated with a conditional ranking table, and CP-networks, directed acyclic networks with local preferences annotated at each vertex. The structures of these networks are similar, but their semantics seem to be quite different. In this paper we discuss if OCF-networks can be used to model the information of CP-networks and vice versa. To answer this question we investigate which restrictions and conditions have to be presupposed to either of the approaches such that one structure can be used to generate the other.

Keywords: CP-network; OCF-network; Acyclic network; Ranking function; Ceteris paribus; Preference; Preferential models

1. Introduction

Representing knowledge, belief or preference as a network rather than, e.g., an exponentially large table of configurations, possible worlds or elementary events allow for spacious and complicated formalisms to become more widely used and successful, the triumph of probabilistics, for instance, is hard to imagine without Bayesian networks [23]. These networks that are well established and successful in probabilistics have been applied to other knowledge representation approaches, too. In the area of semi-quantitative reasoning, for the formalism of ordinal conditional functions (OCF, [25,26]) the approach of OCF-networks [16,7,18,13] has proven to be a lightweight and helpful approach for compact representation of belief states.

On the other hand, directed acyclic networks with local information storage which are, naturally, the core and centre of a Bayesian network, are used in other areas as well. In the area of decision making, the approach of ceteris paribus (CP-) networks models a global preferential relation on the set of worlds based on local preferences [9]. This

* Principal corresponding author.
** Corresponding authors.
E-mail addresses: christian.eichhorn@tu-dortmund.de (C. Eichhorn), matthias.fey@tu-dortmund.de (M. Fey), gabriele.kern-isberner@cs.tu-dortmund.de (G. Kern-Isberner).

http://dx.doi.org/10.1016/j.fss.2016.04.006
0165-0114/

approach also relies on a directed acyclic graph with tables at the vertices that in this case encodes local preferences in the context of parent vertices, so we have a strong resemblance to other network approaches.

If for two network approaches the underlying structures are identical (with respect to the graph-component) or follow similar concepts, such as storing local information on configurations of variables in the context of the configuration of their parent vertices, the question whether these networks are related arises naturally. In this paper we examine whether the structural resemblance between OCF-networks and CP-networks is carried over to formal properties of the formalisms, that is, if both approaches share certain properties. This question is addressed by trying to derive either approach from the other, inspecting if the preferential inferences that can be drawn from both formalisms are identical.

We demonstrate that plainly transferring local preferences between both approaches creates the designated structure. This approach succeeds in generating either structure, but since it just uses local information and does not take the respective global properties, like, for instance, (conditional) independence, into account, it fails to transfer the respective inference behaviour. We use the insights from the plain approach and introduce an approach that allows to construct an OCF-network from every CP-network. We also show that even if we restrict OCF-networks to some extent, there are OCF-networks that cannot be transferred into CP-networks without losing information. With the insights gained from these investigations, we postulate a property for OCF-networks that ensures that the global ranking function is compatible with the local preferences of the network and hence with the CP-network that is generated from the OCF-network by the plain approach. Applied to the generation of OCF-networks from CP-networks this property provides a schema of local ranking tables for OCF-networks that are compatible with the initial CP-network. We present an algorithm that constructs an OCF-network with minimal local ranking values for a CP-network implementing this schema and compare both approaches with respect to their possibility to encode formal properties of the respective other. These results finally give us that OCF-networks are strictly more expressive than CP-networks.

The paper is organised as follows: Section 2 gives the preliminaries necessary for this paper. We then introduce the two approaches of CP-networks (Section 3) and OCF-networks (Section 4) together with the necessary underlying knowledge representation formalisms to an extent that is necessary for the comparison. In the following Section 5 we focus on the question whether CP-networks can be derived from OCF-networks, or vice versa. Here we introduce a plain approach that generates the designated structure but fails to transfer the inference behaviour in the general case in Section 5.1. This problem is addressed in Section 5.2 by restricting OCF-networks to such an extent that local indifference, a concept not compatible with the local information in CP-networks, is excluded and present an algorithm that maps each CP-network to an OCF-network. In Section 6 we use the insights gained by discussing how to mutually derive one network type from the other to compare both approaches. Section 7 then sums up this comparison on basis of the results of the previous sections. We relate our approach to other works in Section 8 and conclude in Section 9.

2. Preliminaries

In this section we introduce the syntax and semantics used in this paper: Let $\Sigma = \{V_1, \ldots, V_m\}$ be a finite propositional alphabet. We denote by v_i the variable V_i in its positive and by $\overline{v}_i$ in its negative outcome, while $\dot{v}_i$ denotes an arbitrary but fixed outcome of V_i. A *literal* is the positive or negative outcome of a variable. The logical language $\mathfrak{L}$ is recursively defined over closure of conjunction ($\wedge$), disjunction ($\vee$) and negation ($\neg$) in the usual way: Every literal is a formula, every negated formula is a formula and if ϕ and ψ are formulas, the conjunction $\phi \wedge \psi$ and disjunction $\phi \vee \psi$ of ϕ and ψ are formulas. For easier reading and shorter formulas, we often omit the connector $\wedge$ and indicate conjunction by juxtaposition of formulas (that is, $\phi\psi$ stands for $\phi \wedge \psi$) and indicate negation by overlining (that is, $\overline{\phi}$ stands for $\neg\phi$).

Interpretations, or *possible worlds* as a syntactical representation of interpretations, are also defined in the usual way; the set of all possible worlds is denoted by Ω. We often use the 1-1 association between worlds and *complete conjunctions*, that is, conjunctions of literals where every variable $V_i \in \Sigma$ appears exactly once. For subsets $\mathbf{A} \subseteq \Sigma$ we refer to the set of *local worlds* by conjunctions of literals where every variable $V_i \in \mathbf{A}$ appears exactly once and denote the set of all local worlds as $\Omega_{\mathbf{A}}$ with individual worlds $\mathbf{a} \in \Omega_{\mathbf{A}}$.

A model ω of a propositional formula $\phi \in \mathfrak{L}$ is a possible world that satisfies ϕ, written as $\omega \models \phi$. The set of all models $\omega \models A$ is denoted by $Mod(A)$. For formulas $\phi, \psi \in \mathfrak{L}$, ϕ *entails* ψ, written as $\phi \models \psi$, iff $Mod(\phi) \subseteq Mod(\psi)$, that is, if and only if for all $\omega \in \Omega$, $\omega \models \phi$ implies $\omega \models \psi$. For sets of formulas $\mathcal{A} \subseteq \mathfrak{L}$ we