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Distributivity between semi-t-operators and semi-nullnorms

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Abstract

Recently the distributivity equation was discussed in families of certain operations (e.g. triangular norms, conorms, uninorms and nullnorms). In this paper we describe the solutions of distributivity between semi-t-operators and semi-nullnorms. Previous results about distributivity between nullnorms can be obtained as simple corollaries.

Keywords: Aggregation operators; Semi-t-operators; Semi-nullnorms; Distributivity equation

1. Introduction

The problem of distributivity has been posed many years ago (cf. Aczél [1], pp. 318–319). A new direction of investigations is mainly concerned of distributivity between triangular norms and triangular conorms ([9], p. 17). Since a short time many authors deal with solution of distributivity equation for aggregation functions [4], fuzzy implications [2], uninorms and nullnorms [14,18], which are generalization of triangular norms and conorms.

Our consideration was motivated by intention of getting algebraic structures which have weaker assumptions than nullnorms or t-operators. A characterization of such binary operations is interesting not only from a theoretical point of view, but also for their applications, since they have proved to be useful in several fields like fuzzy logic framework [11], expert system [13], neural networks [13] or fuzzy quantifiers [11].

First, we introduce weak algebraic structures (Section 2). Then, the distributivity equations are recalled (Section 3). Next, solutions of distributivity equations from described families are characterized (Section 4). Finally, our results are applied to nullnorms, which can be compared with results from [14] and [8] (Section 5).

2. Associative, monotonic binary operations

We start with basic definitions and facts.

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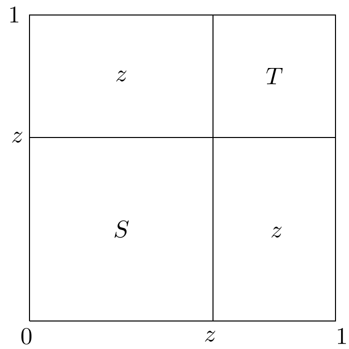

Fig. 1. Structure of nullnorm.

Definition 2.1. (See [10].) A semi-triangular norm T is an increasing, associative operation $T : [0,1]^2 \to [0,1]$ with neutral element 1.

A semi-triangular conorm S is an increasing, associative operation $S : [0,1]^2 \to [0,1]$ with neutral element 0.

A triangular norm T is a commutative semi-triangular norm.

A triangular conorm S is a commutative semi-triangular conorm.

Example 2.2. (See [10].) Well-known triangular norms and triangular conorms are:

$$T_M(x,y) = \min(x,y), \qquad S_M(x,y) = \max(x,y),$$
$$T_P(x,y) = x \cdot y, \qquad S_P(x,y) = x + y - xy,$$
$$T_L(x,y) = \max(x+y-1,0), \qquad S_L(x,y) = \min(x+y,1),$$
$$T_D(x,y) = \begin{cases} \min(x,y), & \text{if } 1 \in \{x,y\} \\ 0, & \text{otherwise,} \end{cases} \qquad S_D(x,y) = \begin{cases} \max(x,y), & \text{if } 0 \in \{x,y\} \\ 1, & \text{otherwise.} \end{cases}$$

Definition 2.3. (See [3].) Operation $V : [0,1]^2 \to [0,1]$ is called nullnorm if it is commutative, associative, increasing, has a zero element $z \in [0,1]$ and that satisfies

$$V(0,x) = x \quad \text{for all } x \le z, \tag{1}$$
$$V(1,x) = x \quad \text{for all } x \ge z. \tag{2}$$

By definition, the case $z = 0$ leads back to triangular norms, while the case $z = 1$ leads back to triangular conorms (cf. [10]). The next theorem shows that it is built up from a triangular norm, a triangular conorm and the zero element.

Theorem 2.4. *(See [3].) Let $z \in (0,1)$. A binary operation V is a nullnorm with zero element z if and only if there exist triangular norm T and triangular conorm S such that (Fig. 1)*

$$V(x,y) = \begin{cases} zS(\frac{x}{z}, \frac{y}{z}) & \text{if } x,y \in [0,z] \\ z + (1-z)T(\frac{x-z}{1-z}, \frac{y-z}{1-z}) & \text{if } x,y \in [z,1] \\ z & \text{otherwise.} \end{cases} \tag{3}$$

Definition 2.5. Element $s \in [0,1]$ is called idempotent element of operation $G : [0,1]^2 \to [0,1]$ if $G(s,s) = s$. Operation G is called idempotent if all elements from $[0,1]$ are idempotent.

Theorem 2.6. *(Cf. [6].) Operation $V : [0,1]^2 \to [0,1]$ is idempotent nullnorm with zero element z if and only if it is given by*

$$V(x,y) = \begin{cases} \max(x,y) & \text{if } x,y \in [0,z] \\ \min(x,y) & \text{if } x,y \in [z,1] \\ z & \text{otherwise.} \end{cases} \tag{4}$$