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On some classes of discrete additive generators

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Abstract

This work is developed in the field of the additive generation of discrete aggregation operators. Specifically, this article deals with the study and applicability of disjunctions that are additively generated by concave, convex and symmetric generators.

Keywords: Discrete binary operation; Disjunction; t-conorm; Additive generator; Symmetric additive generator; S-implication function

1. Introduction

Fuzzy logic is one of the tools for management of uncertainty. In Fuzzy logic we usually work with a continuous scale of certainty values, the real unit interval $[0, 1]$, however implementation restrictions in applications force us to use a finite scale of truth degrees instead of the mentioned continuous one. In this paper we deal with the class of finitely valued disjunction-like operations that contains, in particular, the family of finitely valued t-conorms. In full analogy to the representation theorem of continuous t-conorms, there exists a characterization of smooth (divisible) discrete t-conorms as ordinal sums of Łukasiewicz discrete t-conorms [8,9]. Other references to smooth discrete associative operations are [2–4]. Here our goal is the study of different aspects of the additive generation of a class of discrete binary operations that we call disjunctions; in particular the additive generation of t-conorms (associative disjunctions). Some results related to discrete t-conorms differ substantially from those obtained for ordinary t-conorms defined on $[0, 1]$. In this sense, for instance, we know that a t-conorm with non-trivial idempotent elements is not additively generable; this is not true for discrete t-conorms as we recall in this paper. It seems clear that to develop a theory focused on the additive generation of discrete (associative) disjunctions is useful. Thus, a t-conorm S defined on the scale $\{0, 1, 2, \dots, n\}$ is determined by $\frac{n(n-1)}{2}$ entries. If S admits an additive generator $(0, a_1, \dots, a_n)$ then it can be managed by only n integer values [5,7,10]. In this paper we also point out the usefulness of having discrete additive generators when we have to describe and manage properties of discrete binary operations. Thus, in Section 4, some properties of S-implication functions (Identity, Ordering and Generalized Modus Ponens) are described in terms

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of an additive generator of the t-conorm S, and we state that the symmetry of it is a sufficient condition in order to ensure the fulfillment of those properties.

2. Preliminaries

Consider $L = \{0, 1, \dots, n\}, n \geq 1$, equipped with the usual ordering. We begin recalling basic definitions, examples and properties of finitely-valued t-conorms. A complete exposition of this topic can be found in [9].

2.1. Disjunctions and t-conorms on a finite totally ordered set

Definition 2.1. A *disjunction* on L is a binary operation $D : L \times L \to L$ such that for all $i, i', j, j' \in L$ the following axioms are satisfied:

1. $D(i, j) = D(j, i)$ (commutativity)
2. $D(i, 0) = i$ (boundary condition)
3. $D(i, j) \leq D(i', j')$ whenever $i \leq i', j \leq j'$ (monotonicity)

Note that from these axioms we have $D(i, j) \geq \max\{i, j\}$. In particular, $D(i, n) = n \ \forall i \in L$.

Definition 2.2. A t-conorm on L is an associative disjunction $(D(D(i, j), k) = D(i, D(j, k)), \forall i, j, k \in L)$.

Example 2.3. We can consider as basic t-conorms:

i) the drastic,

$$S_d(i, j) = \begin{cases} i & \text{if } j = 0, \\ j & \text{if } i = 0, \\ n & \text{otherwise}, \end{cases}$$

ii) the maximum $S_M(i, j) = \max\{i, j\}$,

iii) the bounded sum, or Łukasiewicz t-conorm, $S_Ł(i, j) = \min\{i + j, n\}$.

Remark 2.4. The mapping $N(i) = n - i$ is the only strong negation on L ($N : L \to L$ is decreasing and involutive).

Given a t-conorm S, the binary operation $T : L \times L \to L$ defined by $T(i, j) = N(S(N(i), N(j)))$ is a t-norm on L (commutative, associative, increasing in each variable with n as neutral element) called the N-dual of S.

Definition 2.5. A disjunction D on L is *smooth* if

$$0 \leq D(i + 1, j) - D(i, j) \leq 1, \quad \forall i, j \in L, i < n.$$

Definition 2.6. A t-conorm S is Archimedean if for all $i, j \in L \setminus \{0, n\}$ there exists $m \in \mathbb{N}$ satisfying $i_S^{(m)} > j$ where $i_S^{(m)} = S(i_S^{(m-1)}, i)$ and $i_S^{(1)} = i$.

It can be easily proved that a t-conorm on L is Archimedean if and only if the only idempotent elements are 0 and n. This result and the next proposition can be found in [9].

Proposition 2.7. *$S_Ł$ is the only smooth Archimedean t-conorm on L.*

Now, we recall a well-known method for constructing a new t-conorm from two given t-conorms [9].

Proposition 2.8. *Let S_1 be a t-conorm on $\{0, 1, \dots, m\}$ and S_2 a t-conorm on $L = \{0, 1, \dots, n\}$, with $m, n \geq 1$. Consider the binary operation S defined on $\{0, 1, \dots, m + n\}$ as follows:*