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Evidence-theory-based numerical characterization of multigranulation rough sets in incomplete information systems

Anhui Tan [a,b,*], Weizhi Wu [a,b], Jinjin Li [c], Guoping Lin [c]

[a] *School of Mathematics, Physics and Information Science, Zhejiang Ocean University, Zhoushan, Zhejiang, 316022, China*
[b] *Key Laboratory of Oceanographic Big Data Mining and Application of Zhejiang Province, Zhoushan, Zhejiang, 316022, China*
[c] *School of Mathematics and Statistics, Minnan Normal University, Zhangzhou, 363000, Fujian, China*

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Abstract

Multigranulation rough sets are desirable features in the field of rough set, where this concept is approximated by multiple granular structures. In this study, we employ belief and plausibility functions from evidence theory to characterize the set approximations and attribute reductions in multigranulation rough set theory. First, we show that in an incomplete information system, the pessimistic multigranulation approximations can be measured by belief and plausibility functions, whereas the optimistic multigranulation approximations do not possess this characteristic in general. We also give a sufficient and necessary condition for the numerical measurement of optimistic multigranulation approximations by belief and plausibility functions. Second, in an incomplete decision system, the pessimistic multigranulation approximations are also measured by belief and plausibility functions. In the end, an attribute reduction algorithm for multigranulation rough sets is proposed based on evidence theory, and its efficiency is examined by an example. Thus, belief and plausibility functions can be employed to numerically characterize the attribute reductions and to construct an attribute reduction algorithm for multigranulation rough sets.

Keywords: Attribute reduction; Belief function; Evidence theory; Incomplete information system; Multigranulation rough set; Plausibility function

1. Introduction

Rough set theory, which was proposed by Pawlak [17], is a powerful mathematical tool for addressing uncertain knowledge in a wide variety of applications related to machine learning, artificial intelligence, pattern recognition, and decision making [3,11,13,27]. Rough set theory provides a systematic classification mechanism for objects via an equivalence relation. Based on the classification mechanism, the knowledge hidden in information systems may be extracted in the form of decision rules. It is well known that the classical rough set theory based on an equivalence

* Corresponding author. School of Mathematics, Physics and Information Science, Zhejiang Ocean University, Zhoushan, Zhejiang, 316022, China.
E-mail addresses: tananhui86@163.com (A. Tan), wuwz@zjou.edu.cn (W. Wu), jinjinli@mnnu.edu.cn (J. Li), guoplin@163.com (G. Lin).

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relation is still restrictive in terms of its practical applications. To increase the application range for classical rough set theory, many meaningful extensions have been proposed for various circumstances, such as coverings [2,40,28], incomplete databases [9,10], fuzzy sets [6,7], and ordered information systems [34]. It should be noted that classical rough sets and most of the extended rough sets are constructed on only one granular structure, which is generated from a binary relation (a partition or a covering). From this viewpoint, these rough sets are actually single-granulation models. However, in many real applications, a target concept needs to be characterized by multiple granular structures induced by multiple binary relations. To address this issue, Qian et al. [19–21] proposed the concept of multigranulation rough sets (MGRSs), where a target concept is approximated by multiple binary relations on the universe. In MGRSs, different approximation strategies may be employed to deal with special problems according to a user's requirement, i.e., "Seeking commonality while reserving difference" or "Seeking commonality while rejecting difference". Many extensions of MGRSs have been proposed. For instance, Qian et al. [20] extended the rough set model based on a tolerance relation to an incomplete rough set model based on multigranulations. Yang [36] developed a fuzzy MGRS. Xu [34] introduced variable precision MGRSs. Lin et al. [14] introduced neighborhood-based MGRSs and three types of covering-based MGRSs. Huang [8] combined the ideas of MGRSs and intuitionistic fuzzy sets, and developed an intuitionistic fuzzy MGRs. Furthermore, MGRSs have been shown to be useful in many applications. From a multi-granulation viewpoint, Liang et al. [12] designed an efficient feature selection algorithm, which first splits a whole dataset into different small granularities, before obtaining estimates on some selected granularities to yield the reduct of the whole dataset, and then fusing all of the estimates together as an approximate reduct. Qian et al. [18] presented the notion of positive approximation, which aims to granulate data via a sequence of granularities from coarse to fine. Based on the positive approximation, Qian et al. [18] introduced a common accelerator for improving the time efficiency of a heuristic attribute reduction process. She [24] investigated the topological structures of MGRSs from the perspective of multiple granulations.

Similar to rough set theory, evidence theory (also known as Dempster–Shafer theory of evidence) is another theoretical method for uncertainty management [5,23]. The theoretical framework of evidence theory is a belief structure, which comprises a function that maps from the power set of the universe to the unit interval, i.e., a basic probability assignment on the power set, and a family of subsets called focal elements, with associated individual positive weights that sum to one. Based on the belief structure, a pair of set functions on the universe, called belief and plausibility functions, are defined to measure the belief and plausibility degree of uncertain knowledge.

It should to be noticed that there are strong connections between rough set theory and evidence theory, and many studies have addressed this topic [16,25,26,29]. For instance, Yao et al. [39] showed that the belief and plausibility functions can be interpreted by the lower and upper approximation operators in rough set theory. Wu et al. [30–32] associated the belief structure with the rough approximation space and investigated knowledge reductions of Pawlak's rough sets from the viewpoint of evidence theory. Moreover, various extended rough set models are also connected with evidence theory. Chen et al. [4] studied the neighborhood-covering rough sets based on evidence theory, where it was shown that some types of covering approximations can be measured by belief and plausibility functions, whereas some others do not share this characteristic. Xu et al. [33] employed belief and plausibility functions to describe the attribute reductions of ordered information systems. Furthermore, fuzzy rough sets have been studied in the framework of evidence theory [1,22,37,38].

However, as a new direction in rough set theory, MGRS theory has been connected only rarely with evidence theory in previous research. Yan et al. [35] considered the attribute reduction of MGRSs by using belief and plausibility functions. Lin et al. [15] presented an approach for fusing multiple information by combining rough set theory and evidence theory. However, these studies have not constructed the belief structure of MGRSs, thus cannot reveal the numerical characteristic of MGRSs on evidence theory. Moreover, they are still limited in the partitions on the data, which cannot be used for incomplete information. In the present study, we investigate the MGRS theory using belief and plausibility functions from evidence theory. The multigranulation approximations and attribute reductions of MGRSs are both numerically characterized by belief and plausibility functions. Based on these considerations, we establish a basic framework for obtaining the numerical measurements of MGRSs based on evidence theory.

The remainder of this paper is organized as follows. Some related concepts in MGRS theory and evidence theory are reviewed in Sections 2 and 3, respectively. In Section 4, we measure the optimistic and pessimistic multigranulation approximations in incomplete information systems and incomplete decision information systems using the belief and plausibility functions, respectively. In addition, the attribute reductions of MGRSs in incomplete information systems and incomplete decision information systems are numerically characterized by belief and plausibility functions,