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Notes on a comprehensive study of implicator–conjunctor-based and noise-tolerant fuzzy rough sets: Definitions, properties and robustness analysis

Chun Yong Wang

School of Mathematical Sciences, Shandong Normal University, Jinan 250014, PR China

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Abstract

D'eer et al. evaluated the implicator–conjunctor-based fuzzy rough set model or shortly, IC model and different noise-tolerant fuzzy rough set models. However, neither IC model nor Fuzzy Variable Precision Rough Set (FVPRS) model satisfies (ID) property without the reflexivity of fuzzy relation. Upper approximation operator in IC model cannot necessarily be expressed by fuzzy granules. Moreover, a fuzzy $\mathscr{T}$-similarity relation is equivalent to a fuzzy relation that is reflexive and $\mathscr{T}$-Euclidean, where $\mathscr{T}$ denotes a t-norm. Therefore, $\mathscr{T}$-Euclidean condition is redundant for IC model satisfying all properties listed in the work of D'eer et al.

Keywords: Fuzzy rough sets; Implicator–conjunctor-based fuzzy rough set model; Noise-tolerance

1. Introduction

Both fuzzy sets [16] and rough sets [9,10] are effective tools to model and process imperfect data. Dubois and Prade [4] proposed fuzzy rough sets, which combine fuzzy sets and rough sets to deal with fuzziness of concepts and vagueness of information. Moreover, the combinations of fuzzy set theory and rough set theory were further investigated with different fuzzy logical connectives and fuzzy relations in [7,8,11–14] and so on. In [3], D'eer et al. considered these different proposals from the view of the implicator–conjunctor-based fuzzy rough set model or shortly, IC model. Moreover, D'eer et al. evaluated noise-tolerant fuzzy rough sets. However, there are some flaws in [3].

In this paper, we point out that neither IC model nor Fuzzy Variable Precision Rough Set (FVPRS) model [17] satisfies (ID) property without the reflexivity of fuzzy relation. In other words, Propositions 5 and 23 in [3] are correct only if the fuzzy relation is a fuzzy $\mathscr{T}$-preorder instead of a $\mathscr{T}$-transitive fuzzy relation. Upper approximation operator in IC model can be expressed by fuzzy granules only if t-norm is an IMTL-t-norm. We further generalize the conditions under which IC model satisfies (CS) and (UE) properties. Moreover, we point out that a fuzzy $\mathscr{T}$-similarity

E-mail address: chunyong_wang@163.com.

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is equivalent to a fuzzy relation that is reflexive and $\mathscr{T}$-Euclidean. Therefore, $\mathscr{T}$-Euclidean condition in Propositions 6 and 8 is redundant. We present a concise version of Propositions 6 and 8 in [3].

The content of the paper is organized as follows. In Section 2, we recall some fundamental concepts and related properties of fuzzy logical connectives and fuzzy sets. Section 3 further discusses the properties of IC and noise-tolerant fuzzy rough set models. In the final section, we present some conclusions of our research.

2. Preliminaries

In this section, we present some basic concepts and terminology used throughout the paper.

2.1. Fuzzy logical connectives

A mapping $\mathscr{C} : [0, 1]^2 \to [0, 1]$ is called a *conjunctor* if it is increasing in both arguments and satisfies the boundary conditions $\mathscr{C}(0, 0) = \mathscr{C}(0, 1) = \mathscr{C}(1, 0) = 0$ and $\mathscr{C}(1, 1) = 1$. It is called a *border conjunctor* if $\mathscr{C}(1, x) = x$ holds for all $x \in [0, 1]$. A commutative and associative border conjunctor is called a *t-norm* and denoted as $\mathscr{T}$.

A mapping $\mathscr{D} : [0, 1]^2 \to [0, 1]$ is called a *disjunctor* if it is increasing in both arguments and satisfies the boundary conditions $\mathscr{D}(1, 1) = \mathscr{D}(0, 1) = \mathscr{D}(1, 0) = 1$ and $\mathscr{D}(0, 0) = 0$. It is called a *border disjunctor* if $\mathscr{D}(0, x) = x$ holds for all $x \in [0, 1]$. A commutative and associative border disjunctor is called a *t-conorm* and denoted as $\mathscr{S}$.

A function $\mathscr{N} : [0, 1] \to [0, 1]$ is called a *negator* if it satisfies $\mathscr{N}(0) = 1$ and $\mathscr{N}(1) = 0$. A negator $\mathscr{N}$ is said to be *involutive* if $\mathscr{N}\big(\mathscr{N}(x)\big) = x$ holds for all $x \in [0, 1]$. The involutive negator $\mathscr{N}(x) = 1 - x$ for all $x \in [0, 1]$ is usually referred as the *standard negator* and denoted as $\mathscr{N}_s$.

Let $\mathscr{C}$, $\mathscr{D}$ and $\mathscr{N}$ be a conjunctor, a disjunctor and an involutive negator, respectively. Then the $\mathscr{N}$-dual of $\mathscr{C}$ is a disjunctor $\mathscr{D}_{\mathscr{C},\mathscr{N}}$ and the $\mathscr{N}$-dual of $\mathscr{D}$ is a conjunctor $\mathscr{C}_{\mathscr{D},\mathscr{N}}$ defined by, respectively,

$$\mathscr{D}_{\mathscr{C},\mathscr{N}}(x, y) = \mathscr{N}\big(\mathscr{C}(\mathscr{N}(x), \mathscr{N}(y))\big) \text{ and } \mathscr{C}_{\mathscr{D},\mathscr{N}}(x, y) = \mathscr{N}\big(\mathscr{D}(\mathscr{N}(x), \mathscr{N}(y))\big) \text{ for all } x, y \in [0, 1].$$

Similarly, the $\mathscr{N}$-dual of a t-norm is a t-conorm and vice versa.

A mapping $\mathscr{I} : [0, 1]^2 \to [0, 1]$ is called an implicator if it satisfies the boundary conditions according to Boolean implicator, and is decreasing in the first and increasing in the second argument. An implicator is called a *border implicator* if $\mathscr{I}(1, x) = x$ holds for all $x \in [0, 1]$. Moreover, an implicator $\mathscr{I}$ is said to satisfy *weak confinement principle*, if $x \leqslant y \Rightarrow \mathscr{I}(x, y) = 1$ for all $x, y \in [0, 1]$, which is also called a *WCP-implicator* in this paper. The induced negator $\mathscr{N}_{\mathscr{I}}$ of an implicator $\mathscr{I}$ is defined as $\mathscr{N}_{\mathscr{I}}(x) = \mathscr{I}(x, 0)$ for all $x \in [0, 1]$.

Let $\mathscr{C}$, $\mathscr{D}$ and $\mathscr{N}$ be a border conjunctor, a border disjunctor and a negator, respectively. Then an implicator $\mathscr{I}$ is called

- an S-implicator $\mathscr{I}_{\mathscr{D},\mathscr{N}}$ based on $\mathscr{D}$ and $\mathscr{N}$ if $\mathscr{I}_{\mathscr{D},\mathscr{N}}(x, y) = \mathscr{D}\big(\mathscr{N}(x), y\big)$ for all $x, y \in [0, 1]$;
- an R-implicator $\mathscr{I}_{\mathscr{C}}$ based on $\mathscr{C}$ if $\mathscr{I}_{\mathscr{C}}(x, y) = \sup\big\{z \in [0, 1] | \mathscr{C}(x, z) \leqslant y\big\}$ for all $x, y \in [0, 1]$.

Notice that S-implicators are border implicators. Moreover R-implicators are both border implicators and WCP-implicators. An implicator $\mathscr{I}$ is said to be an *IMTL-implicator* [5,6], if $\mathscr{I}$ is based on a left-continuous t-norm and has an involutive induced negator. A left-continuous t-norm of which the R-implicator is an IMTL-implicator, is called an *IMTL-t-norm*.

Let $\mathscr{I}$ be an implicator and $\mathscr{N}$ an involutive negator. Then the induced conjunctor $\mathscr{C}_{\mathscr{I},\mathscr{N}}$ of $\mathscr{I}$ and $\mathscr{N}$ is defined as $\mathscr{C}_{\mathscr{I},\mathscr{N}}(x, y) = \mathscr{N}\big(\mathscr{I}(x, \mathscr{N}(y))\big)$ for all $x, y \in [0, 1]$, which is not necessarily a t-norm. Moreover, the $\mathscr{N}$-dual of $\mathscr{I}$ is defined as $\mathscr{J}_{\mathscr{I},\mathscr{N}}(x, y) = \mathscr{N}\big(\mathscr{I}(\mathscr{N}(x), \mathscr{N}(y))\big)$ for all $x, y \in [0, 1]$, which is called a *coimplicator* in [2].

2.2. Fuzzy sets and fuzzy relations

Let U be a universe. Then a mapping $A : U \to [0, 1]$ is called a *fuzzy set*. The family of all fuzzy sets on U is denoted as $\mathscr{F}(U)$. Given $\alpha \in [0, 1]$, a fuzzy set $A \in \mathscr{F}(U)$ is a constant (fuzzy) set, if $A(x) = \alpha$ for all $x \in U$, denoted as $\widehat{\alpha}$.