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# Properties of Raha's similarity-based approximate reasoning method

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**Abstract**

A similarity-based approximate reasoning methodology that requires the construction of fuzzy relations was proposed by Raha et al. [26]. This paper investigates properties of four inferred conclusions calculated by Raha's similarity-based approximate reasoning method. The relationships among the four inferred conclusions are examined. The monotonicity and the approximation property for similarity-based approximate reasoning methods are defined. Then the monotonicity and the approximation property of Raha's similarity-based approximate reasoning method are studied.


*Keywords:* Similarity measure; Approximate reasoning; Implication function; t-Norm; Fuzzy equivalence; Fuzzy relation

## 1. Introduction

As an approach to describing the similarity degree between two fuzzy sets, the concept of similarity measure has gained successful applications in various fields. Many researchers have contributed to the comparative study of similarity measures of fuzzy sets. They discussed similarity measures of fuzzy sets not only in theory [3,6,8,11,14,15,18–22,31,32] but also in application [4,5,7,12,26,27,29,34,36]. The widest and typical application is found in approximate reasoning [23–27,29].

In [29], Turksen and Zhong commented and showed that the notion of similarity measure can be successfully applied in approximate reasoning. They proposed a similarity-based approximate reasoning method called approximate analogical reasoning schema. It is shown that the method is applicable to both point-valued and interval-valued fuzzy sets. In [4,5], Chen proposed two similarity-based approximate reasoning methods for medical diagnosis problems. Chun [7] presented a similarity-based bidirectional approximate reasoning method which can express the decision maker's disposition. Yeung and Tsang [36] analyzed and compared six similarity-based approximate reasoning methods. Wang et al. [34] introduced a fuzzy similarity reasoning method and put it into the framework of fuzzy logic by using partial valuation.

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In the above-mentioned work, the similarity-based approximate reasoning method does not require the construction of fuzzy relations. The membership value of each element of the consequent fuzzy set of the rule is modified directly by the similarity degree between the fact and the antecedent of a rule. In [26], Raha et al. proposed a similarity-based approximate reasoning method in which the construction of fuzzy relations is required. The method is given by means of an algorithm including four steps. In the proposed algorithm, they first express the conditional statement as a fuzzy relation and interpret it as a conditional fuzzy relation $R(A, B)$. Then they compute the similarity degree $N(A, A^*)$ between the fact $A^*$ and the antecedent of a rule $A$ and use $N(A, A^*)$ to modify $R(A, B)$. The modification is based on a scheme and the result is interpreted as a modified conditional relation $R(A, B \mid A^*)$. Finally, by using the sup-projection operation on $R(A, B \mid A^*)$, an inferred conclusion is obtained.

In Raha's work, for modelling the conditional fuzzy relation, two models of fuzzy rules were given (i.e., implication-based and conjunction-based [9,10]). Two schemes to compute modified conditional relations were proposed and thus two kinds of modified conditional relations were obtained. In [12], Feng and Liu proposed a novel approach to similarity-based approximate reasoning in an interval-valued fuzzy environment. It is shown that the primary mechanism of Feng's approach is similar to Raha's approach except that a different scheme is used to calculate the modified conditional relation. Two models of fuzzy rules were also studied in Feng's work. Hence, two kinds of modified conditional relations were obtained. By using the sup-projection operation on the proposed modified conditional relations, Raha et al. [26] and Feng and Liu [12] respectively found that the implication-based model would lead to completely non-specificity. Therefore, both of them preferred the conjunction-based model. Actually, the implication-based model can also be used in Raha's and Feng's approaches on condition that the inf-projection operation is used on modified conditional relations. Hence, besides the inferred conclusions obtained in Raha's work (denoted by ${B_1}^*$) and Feng's work (denoted by ${B_3}^*$), other two inferred conclusions (denoted by ${B_2}^*$ and ${B_4}^*$) are obtained in the present paper.

It is shown that Feng's conclusion ${B_3}^*$ can be calculated by Raha's approach when Feng's scheme is used in Step 3 of Raha's algorithm [26]. The conclusions ${B_2}^*$ and ${B_4}^*$ are respectively obtained from the modified conditional relations proposed by Raha et al. [26] and Feng and Liu [12]. Therefore, the above-mentioned four inferred conclusions can be obtained through Raha's algorithm. In this sense, all of them can be seen as conclusions of Raha's similarity-based approximate reasoning method.

In order to make distinctions between two models for modelling the conditional fuzzy relation and study what modifications of $R(A, B)$ by $N(A, A^*)$ make sense or not in similarity-based approximate reasoning, this paper investigates properties of the four inferred conclusions calculated by Raha's similarity-based approximate reasoning method. The relationships among the four inferred conclusions are studied. The monotonicity and the approximation property for similarity-based approximate reasoning methods are defined. The monotonicity and the approximation property of Raha's similarity-based approximate reasoning method are studied. The obtained results may provide us with a criterion to select a suitable inferred conclusion in similarity-based approximate reasoning.

In Section 2 we present some basic notations and definitions of fuzzy connectives and similarity measure. In Section 3 we review Raha's and Feng's approaches to similarity-based approximate reasoning and obtain two additional inferred conclusions by using Raha's and Feng's modified conditional relations. Section 4 includes three subsections. In the first subsection, we investigate relationships among four inferred conclusions. In the second subsection, we discuss the monotonicity of Raha's similarity-based approximate reasoning method. In the third subsection, we study the approximation property of Raha's similarity-based approximate reasoning method. Finally we conclude our main results in Section 5.

## 2. Preliminaries

In this section we brief recall, without proof, some preliminary definitions and results.

Throughout this paper, $X$ and $Y$ are the universes of discourse; $F(X)$ and $F(Y)$ are the classes of all fuzzy subsets of $X$ and $Y$, respectively; $A(x)$ is the membership function of $A \in F(X)$; $A'$ is the complement of $A$ with $A'(x) = 1 - A(x)$ for all $x \in X$.

**Definition 1.** (See [16].) An associative, commutative and increasing function $T : [0, 1]^2 \longrightarrow [0, 1]$ is called a t-norm if it has the neutral element equal to 1. A t-norm $T$ is left-continuous if $T(u, \bigvee_{j\in J} v_j) = \bigvee_{j\in J} T(u, v_j)$ holds for an arbitrary index set $J$ and for each $u \in [0, 1]$ and $(v_j)_{j\in J} \in [0, 1]^J$.