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Model checking fuzzy computation tree logic

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Abstract

Traditional temporal logics such as linear temporal logic and computation tree logic are widely used to specify properties of reactive systems. Model checking is a well-established technique for verifying if a desired property described as a temporal logic formula holds over a reactive system model. This paper presents fuzzy computation tree logic (FCTL), a fuzzy extension of temporal logics, by combining general fuzzy logic with computation tree logic, and discusses its model checking problem. First, the notion of fuzzy Kripke structures (FKSs) and the syntax and semantics of their specification language FCTL are introduced. Then we give a direct model checking algorithm for FCTL over FKS. On the other hand, we study when FCTL model checking problem can be reduced to classical model checking ones, and give a reduction method for an important fragment of FCTL.

Keywords: Model checking; Temporal logic; Computation tree logic; Fuzzy logic; Triangular norm

1. Introduction

Temporal logics [2,15], like linear temporal logic (LTL), computation tree logic (CTL) and μ-calculus, are a useful formalism for specification of reactive systems such as operating systems, schedulers, and discrete-event controllers. They have been successfully used in many situations, especially for model checking [2,15] – the automatic verification that a finite-state model of a system satisfies a temporal logic specification.

Classical temporal logics are limited to dealing with crisp assertions, where the associated semantics are crisp. However, it is possible that assertions encountered in the real world are not sufficiently precise, and cannot be simply treated in terms of yes–no questions. For example, in the case where computation is increasingly intertwined with sensor-derived perceptions of the physical world, the states of a system may become uncertain and cannot be described by precise values since sensors for a state observer are often inaccurate.

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The methodology of fuzzy logics is a good tool for dealing with imprecision, uncertainty, and vagueness. Much effort has been made to extend temporal logics with fuzzy logics to provide the ability of dealing with vague knowledge. Mukherjee and Dasgupta [36] proposed a fuzzy real-time temporal logic FRTL, presented an algorithm for monitoring FRTL properties on finite traces, and showed an application of FRTL to the quality quantification of embedded control systems. Moon et al. [35] introduced the fuzzy branching temporal logic FBTL to model dynamic systems with uncertain temporal information. More recently, FBTL has been chosen as the semantics of requirement specification language RELAX [40]. Other works on fuzzy temporal logics include [10,30,32,43].

In fuzzy logic, classical Boolean conjunction, disjunction, implication, and negation are extended to triangular norms (t-norms), triangular conorms (t-conorms), implication functions, and negation functions, respectively. There are a lot of families of fuzzy logics. The most important ones are Zadeh, Łukasiewicz, Gödel, and Product fuzzy logics. From a semantics point of view, the previous works on fuzzy temporal logics mainly rely on Zadeh fuzzy logic. There are few works that consider temporal logics from the view of the other fuzzy logics except [21,22,26]. Multi-valued model checking [11,12] is a multi-valued extension of classical model checking to an arbitrary distributive De Morgan algebra. By instantiating the distributive De Morgan algebra to the closed unit interval [0, 1], the results on multi-valued model checking can be naturally transferred to temporal logics based on Zadeh fuzzy logic.

The aim of this paper is to consider CTL under general fuzzy logics and solve its model checking problem. We first introduce the model of fuzzy Kripke structure (FKS), which can be used as a general formalism for fuzzy systems. FKSs can be regarded as a state-analogue to fuzzy automata [8,13,14,17,31,38]. To formulate the properties of FKSs, we present a branching-time temporal logic – fuzzy computation tree logic (FCTL) – as a specification language of FKSs. The syntax of FCTL is the same as that of CTL. We give the semantics of FCTL over FKSs in the sense of fuzzy logics. Such a semantics is partly motivated by the semantics of fuzzy description logics [3,4,24].

The multi-valued model checking problems can be solved either using direct model checking algorithms on the multi-valued Kripke structures [32,39] or giving a reduction to several classical model checking problems [6,23]. Following the methodology of the multi-valued model checking, we approach the FCTL model checking problem over the FKSs in two ways. For the direct model checking algorithm for FCTL over FKSs, we first provide the value iteration method that achieves polynomial time complexity in the size of the system, and then based on the ideas, we give an improved algorithm whose complexity is linearithmic in the size of the system. For the reduction method for FCTL model checking problem, we first give the necessary conditions for that FCTL model checking problem can be reduced to classical CTL model checking problem. By a counterexample, we show the usual reduction method for FCTL based on Zadeh fuzzy logic is not applicable to FCTL based on arbitrary fuzzy logic. Hence we have to give a new reduction method to solve the problem. Such a method only works well for an important fragment of FCTL; we leave the whole FCTL as a future work.

Related work Some related studies should be distinguished here. The most related one is model checking computation tree logic over Markov chains, which is called probabilistic computation tree logic (PCTL) [2,25]. In comparison with the works on model checking PCTL, we note that: On the one hand, the syntax of FCTL is different from that of PCTL. The former is the same as that of classical CTL, while the latter replaces the CTL universal and existential path quantifications with the probabilistic operator $\mathbb{P}_J$, where J is an interval in [0, 1]. Moreover, the semantics of FCTL coincides with that of CTL when FCTL is defined over the Kripke structure, while the CTL formula $\forall\varphi_1 U \varphi_2$ cannot be expressed in PCTL. On the other hand, the time complexities of algorithms for model checking FCTL and PCTL are different: For finite Markov chain $\mathcal{M}$ and PCTL formula φ, the PCTL model checking problem can be solved in time $O(|S| \cdot |\mathcal{M}| \cdot |\varphi|)$, where $|S|$ is the numbers of states in $\mathcal{M}$, $|\mathcal{M}|$ is the size of $\mathcal{M}$, and $|\varphi|$ is the length of φ, while the time complexity of the algorithm of FCTL model checking is linearithmic in the size of the system.

Another interesting work is [33], in which computation tree logic based on possibility measures called possibilistic CTL (PoCTL) is explored. The syntax of PoCTL is the same as that of PCTL except that the possibilistic operator Po_J replaces the probabilistic operator $\mathbb{P}_J$. The algorithm of model checking PoCTL and its time complexity are similar to those of model checking PCTL over Markov chains, respectively. Roughly speaking, the semantics of PoCTL is based on Zadeh fuzzy logic.

Note the fragment of FCTL, FCTL($\exists X, \forall X$), is closely related to many-valued (fuzzy) modal logics (see, for example, [5,20,41], and the references therein). The relationship between the many-valued modal logic based on finite Heyting algebra and the classical modal logic has been addressed in [20], where the semantics of $\forall X$ in fuzzy modal