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Short communication

A comment on "Construction of fuzzy automata from fuzzy regular expressions"

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Abstract

The aim of this comment is to point out that the fuzzy language recognized by a nondeterministic finite automaton, which is associated with a fuzzy regular expression, in the context of Stamenković–Ćirić's method (Stamenković and Ćirić (2012) [11]), is recognized by a fuzzy finite automaton with ε-moves. Every fuzzy automaton with ε-moves is also equivalent to a fuzzy automaton and there are effective methods for removing ε-moves in order to obtain the equivalent fuzzy automaton. In this way, our proposal to convert a fuzzy regular expression to a fuzzy finite automaton is based on the construction of a fuzzy automaton with ε-moves for the fuzzy regular expression and the construction of an equivalent fuzzy finite automaton by means of ε-removal operation.

Keywords: Fuzzy automata; Fuzzy automata with ε-moves; ε-Removal operation; Fuzzy regular expressions; Position automata; Lattice-ordered monoids

1. Introduction

In automata theory, the computation of a finite automaton from a regular expression is a problem which has been tackled many times in the past. The classical method, Thompson's construction [13], creates a finite automaton with a number of states and transitions linear in the length of the regular expression. But Thompson's construction builds nondeterministic finite automata with transitions by the empty word (ε-moves). Other alternative techniques have been introduced to create finite automata without ε-moves representing regular expressions. For example, the *position automaton* [2,7] of a regular expression has exactly $n + 1$ states where n is the number of occurrences of alphabet symbols appearing in the regular expression. Position automata are more adequate than Thompson's automata for some applications based on regular expression search [9].

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The concept of *fuzzy finite automaton* was introduced in the very early age of fuzzy set theory [10,4,14,12]. In the modern formulation of a fuzzy finite automaton, state transitions, initial and final states take truth values from certain algebraic structures. In this paper, we consider fuzzy finite automata with membership values in an integral lattice-ordered monoid (ℓ-monoid) [5], denoted as $\mathcal{L}$. Li and Pedrycz [5] have proved that any fuzzy language over $\mathcal{L}$ can be represented by a fuzzy regular expression if and only if it can be recognized by a fuzzy finite automaton (over $\mathcal{L}$). The reason to consider an integral ℓ-monoid as the underlying structure of truth values is mainly due to the fact that *for any fuzzy language over $\mathcal{L}$, the Kleen closure is well defined* [5].

In [6], Z. Li et al. introduce fuzzy finite automata with ε-moves with membership values over an ℓ-monoid. In such a paper, they show the equivalence between a fuzzy finite automaton with ε-moves M and a fuzzy finite automaton N by using the transitive closure of a fuzzy relation on the states of M. This fuzzy relation represents all possible ε-moves of M.

Therefore, it seems reasonable to extend Thompson's construction to convert a fuzzy regular expression to a fuzzy finite automaton with ε-moves. Furthermore, it can be converted to an equivalent fuzzy finite automaton by removing ε-moves [8]. This extension of Thompson's construction is very simple and does not provide new insights in fuzzy automata theory.

More recently, Stamenković and Ćirić [11] have provided an elegant method for constructing an equivalent fuzzy finite automaton from a given fuzzy regular expression. We briefly explain their method for an integral ℓ-monoid $\mathcal{L} = (L, \wedge, \vee, \otimes, 0, 1)$:

Let α be a fuzzy regular expression over a finite alphabet X. They define an ordinary regular expression α_R over a new alphabet $X \cup Y$, where Y consists of the symbols associated with different truth values appearing in α. They define a mapping φ in order to map all symbols from X to 1, and symbols from Y to the related truth values appearing in α. Then, they provide a homomorphism φ_α^* of the free monoid $(X \cup Y)^*$ to the monoid $(L, \otimes, 1)$. By using φ_α^*, they establish a relationship between the fuzzy language $\|\alpha\|$ represented by α and the crisp language $\|\alpha_R\|$ represented by α_R. Starting from any nondeterministic finite automaton A, which recognizes the language $\|\alpha_R\|$, they construct a fuzzy finite automaton A_α^r such that (i) it recognizes the fuzzy language $\|\alpha\|$, and (ii) it is a reduced version of the automaton A. However, the construction of A_α^r requires the following steps:

1. Define a reflexive and transitive fuzzy relation R_A on the set of states of A. R_A is expressed in terms of φ_α^* and the transition relation of A.
2. Build a fuzzy finite automaton A_α. The fuzzy transition relation and the fuzzy set of final states of A_α are expressed in terms of the fuzzy relation R_A, the transition relation of A, and the set of final states of A.
3. Define the reduced set of states of A_α^r by using the states of A_α and the transition relation of A.
4. Build the reduced fuzzy finite automaton A_α^r by using A_α and the reduced set of states of A_α^r.

The steps presented above resemble to the method of converting a fuzzy finite automaton with ε-moves to a fuzzy finite automaton [6]. In particular, steps 1. and 2. require up to $(2 \times |X| + n) \times n^2$ multiplications ($\otimes$ operations) where n is the number of states of A and $|X|$ is the cardinal of the alphabet X (see [11, Lemma 4.2 and Theorem 4.4]).

As ε-removal operation can be efficiently implemented by an $\mathcal{O}(n^2)$ algorithm [8], we study the practicality of fuzzy automata with ε-moves within the context of Stamenković–Ćirić's method. The key point of our research is to prove that the fuzzy language recognized by a nondeterministic finite automaton via a homomorphism φ^* is recognized by a fuzzy finite automaton with ε-moves. This fuzzy finite automaton with ε-moves has the same number of states that the given nondeterministic automaton. In this way, our proposal to convert a fuzzy regular expression α to a fuzzy finite automaton is as follows:

Starting from any nondeterministic finite automaton A, which recognizes the language $\|\alpha_R\|$, our method requires two steps:

1. Build directly from A and the mapping φ, a fuzzy automaton with ε-moves M_α which recognizes the fuzzy language $\|\alpha\|$; and
2. Build an equivalent fuzzy finite automaton N_α by applying the ε-removal operation on M_α.