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Sugeno integrals with respect to level dependent capacities

Peter Smrek

Slovak University of Technology in Bratislava, Faculty of Civil Engineering, Radlinského 11, 81005 Bratislava, Slovakia

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Abstract

The standard Sugeno integral has several equivalent ways to be introduced. This equivalence fails when generalizing the standard capacities into level dependent capacities. We discuss several possible types of Sugeno integral based on level dependent capacities. An illustrative example is added. Relations between different types of level dependent capacities-based Sugeno integrals are presented, showing that there are exactly three different types of studied integrals.

Keywords: Capacity; Level dependent capacity; Sugeno integral

1. Introduction

Sugeno has introduced his integral in 1974 [10].

Definition 1. Let $(X, \mathcal{A})$ be a measurable space. A monotone set function $m : \mathcal{A} \to [0, 1]$ is called a capacity whenever $m(\emptyset) = 0$ and $m(X) = 1$.

Fix an arbitrary measurable space $(X, \mathcal{A})$ and denote by $\mathcal{F}$ the class of all $\mathcal{A}$-measurable functions $f : X \to [0, 1]$. If m is a capacity, then Sugeno integral is a functional $Su_m(f) : \mathcal{F} \to [0, 1]$, given by

$$Su_m(f) = \sup\{\min(a, m(A)) \mid a \cdot 1_A \le f\}. \tag{1}$$

Equivalently, Su_m can be expressed as

$$Su_m(f) = \sup\{\min(a, m(f \ge a)) \mid a \in [0, 1]\}, \tag{2}$$

or

$$Su_m(f) = \sup\{\min(m(A), \inf\{f(x) \mid x \in A\} \mid A \in \mathcal{A})\}. \tag{3}$$

E-mail address: peter.smrek@stuba.sk.

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In [4], see also [1], another equivalent definition of Sugeno integral was introduced, namely

$$Su_m(f) = \inf\{\max(a, m(f \geq a)) \mid a \in [0,1]\}. \tag{4}$$

Recently, the concept of capacities was extended to level dependent capacities in [2], see also [5,6].

Definition 2. Let $(X, \mathcal{A})$ be a measurable space. A mapping $M : \mathcal{A} \times [0,1] \to [0,1]$ such that for each $t \in [0,1]$, $M(\cdot, t) = m_t$ is a capacity, is called a level dependent capacity.

The aim of this paper is to discuss the Sugeno integral with respect to level dependent capacities, discussing its different forms based on extension of formulae (1)–(4) and to find possible equivalencies or inequalities for these new forms.

The paper is organized as follows. In Section 2, we introduce extremal forms of the level dependent capacities based Sugeno integral following the approach from [5,6], and versions of this integral deduced from formulae (1)–(4), as well as a Min-copula based form. Some trivial relations for the different versions of the level dependent capacities based Sugeno integrals are presented. In Section 3 an illustrative example is added. Next, in Section 4, we refine the relations of all types of introduced level dependent capacities based Sugeno integrals showing that, in fact, there are exactly three different types of level dependent capacities based Sugeno integrals. Finally, some concluding remarks are added.

2. Sugeno integral and level dependent capacities

Sugeno integral, as introduced in [10], is a special instance of universal integrals proposed by Klement et al. in [7]. In the framework of universal integrals, all information contained in a capacity m and a fuzzy event f (i.e., a measurable function $f : X \to [0,1]$) is summarized into one special function $h_{m,f} : [0,1] \to [0,1]$ given by $h_{m,f}(t) = m(f \geq t) = m(\{x \in X \mid f(x) \geq t\})$. This function can be seen as a generalized survival function (i.e., a complement to the distribution function). In the case of universal integrals extended for level dependent capacities, Klement at al. have proposed in [5] to consider the function $h_{M,f} : [0,1] \to [0,1]$ given by $h_{M,f}(t) = M(\{f \geq t\}, t) = m_t(f \geq t)$. Observe that while $h_{m,f}$ is a decreasing function (and thus Borel measurable), these properties need not be satisfied for $h_{M,f}$. Then, again following [5,6], two decreasing boundaries $(h_{M,f})_*$, $(h_{M,f})^* : [0,1] \to [0,1]$ of $h_{M,f}$ can be considered,

$$(h_{M,f})_* = \sup\{h : [0,1] \to [0,1] \mid h \text{ is decreasing}, h \leq h_{M,f}\}$$

and

$$(h_{M,f})^* = \inf\{h : [0,1] \to [0,1] \mid h \text{ is decreasing}, h \geq h_{M,f}\}.$$

It is not difficult to check that for each $t \in [0,1]$ it holds [5]

$$(h_{M,f})_*(t) = \inf\{h_{M,f}(u) \mid u \in [0,t]\} \text{ and}$$
$$(h_{M,f})^*(t) = \sup\{h_{M,f}(v) \mid v \in [t,1]\}.$$

Obviously $(h_{M,f})_* = h_{M,f} = (h_{M,f})^*$ if and only if $h_{M,f}$ is decreasing. Following [5,6], the smallest Sugeno integral based on level dependent capacities $(Su_M)_* : \mathcal{F} \to [0,1]$ is given by

$$(Su_M)_*(f) = \sup\{\min(t, (h_{M,f})_*(t)) \mid t \in [0,1]\}. \tag{5}$$

Similarly, the greatest Sugeno integral based on level dependent capacities $(Su_M)^* : \mathcal{F} \to [0,1]$ is given by

$$(Su_M)^*(f) = \sup\{\min(t, (h_{M,f})^*(t)) \mid t \in [0,1]\}. \tag{6}$$

Evidently it holds

$$(Su_M)^*(f) = \sup\{\min(t, h_{M,f}(v)) \mid 0 \leq t \leq v \leq 1\} \tag{7}$$

and

$$(Su_M)_*(f) = \sup\{\min(t, h_{M,f}(u)) \mid 0 \leq u \leq t \leq 1\}. \tag{8}$$