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Some implications of the restricted distributivity of aggregation operators with absorbing elements for utility theory

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Abstract

The issue of restricted distributivity (conditional distributivity), i.e., distributivity on the relaxed domain, is crucial for many different areas such as utility theory and integration theory. This paper considers restricted distributivity of a continuous nullnorm with respect to a continuous t-conorm and some applications of the obtained results in utility theory.

Keywords: Absorbing element; Nullnorms; Restricted distributivity; Utility function; Threshold

1. Introduction

Aggregation operators play an important role in many different theoretical and practical fields (fuzzy set theory, theory of optimization, operations research, information theory, engineering design, game theory, voting theory, integration theory, etc.), particularly in various approaches to the problem of decision making (see [2,12,13,18]). The classical approach to decision making under uncertainty assumes that uncertainty is represented by probability distributions. This approach corresponds to the well-known classical utility theory that is based on the notion of mathematical expectation. Its axiomatic foundations, that were given by Von Neumann and Morgenstern [21], rely on the notion of probabilistic mixtures [14]. In order to generalize decision theory to non-probabilistic uncertainty, the approach focused on the generalized mixture sets emerged as a highly acceptable answer. In [7] Dubois et al. have extended the notion of mixtures to decomposable (pseudo-additive) measures (see [11,22,25]) that include some well-known set functions such as probability and possibility measures [26]. In axiomatic foundations of generalized mixtures crucial role play pairs of t-norms and t-conorms that satisfy the distributivity law. This fact has led to new kind of mixtures called possibilistic mixtures, that form the basis of the possibilistic utility theory (see [8]).

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Based on the characterization of pairs of continuous t-norms and t-conorms that satisfy the distributivity law on the relaxed domain [18], Dubois, Pap, Prade in [9] have shown that beyond the possibilistic and probabilistic mixtures, the only remaining possibility is hybridization, i.e., a mixture is possibilistic under a certain level a, $0 \leq a \leq 1$, and probabilistic above it. The hybrid utility function by means of hybrid mixtures was given in [9]. It should be stressed that one of the closely related problems is the problem of independence of events for decomposable measures. Dubois, Pap, Prade in [9,10] have shown that the same condition, i.e., relaxed distributivity, must be satisfied between the t-conorm characterizing the pseudo-additive measures and the t-norm expressing independence. Mixtures and generalized independence (called separability) are closely linked within utility theory (see [6,9]). Namely, this connection allows the construction of an algorithm for calculating the value of measures in the compound lottery, and hence the algorithm for calculating the hybrid mixture. Also, an interesting application of this conditional distributivity on two Borel–Cantelli lemmas and independence of events for decomposable measures was given in [5].

Therefore, it is obvious that the special pairs of aggregations operators that satisfy distributivity law are highly useful in utility theory. The aim of this paper is to extend on the previous research from [9] towards aggregation operators with non-trivial absorbing element (annihilator), that can be applicable in utility theory for modeling behavior of a decision maker. Thus, the focus is now on the distributivity equation

$$F(x, S(y, z)) = S(F(x, y), F(x, z)), \quad x, y, z \in [0, 1], \quad S(y, z) < 1,$$

where F is a continuous nullnorm and S is a continuous t-conorm and consequences its solutions to utility theory. This paper is based on [16].

The paper is organized as follows. Section 2 contains preliminary notions concerning nullnorms, restricted distributivity and hybrid utility function. Properties of the utility function U_F based on a continuous nullnorm F with the non-trivial absorbing element k are given in the third section. Section 4 is devoted to the analysis of behavior of the decision maker with respect to the utility function U_F. Some concluding remarks are given in the fifth section.

2. Preliminaries

A short overview is first given in this section regarding some of the basic notions that are essential for this topic (see [4,9,13,18,19]).

2.1. Aggregation operators

First, let us recall the basic definition of an aggregation operator on [0, 1].

Definition 1. (See [13].) An aggregation operator is a function $A^{(n)} : [0, 1]^n \to [0, 1]$ that is non-decreasing in each variable and that fulfills the following boundary conditions

$$A^{(n)}(0, \dots, 0) = 0 \quad \text{and} \quad A^{(n)}(1, \dots, 1) = 1.$$

Of course, the previous definition of aggregation operators can be extended to an arbitrary real interval $[a, b]$. The integer n represents the number of input values of the observed aggregation operator. Since the topic of this paper are the binary aggregation operators, they will be denoted simply by A instead of $A^{(2)}$. Many additional properties such as continuity, associativity, commutativity, idempotency, decomposability, autodistributivity, bisymmetry, existence of neutral and absorbing elements, etc., are often required for aggregation operators, depending on the background in which the aggregation is performed (see [13]).

The first type of aggregation operators that will be used in this paper is the aggregation operator with an absorbing element, namely the nullnorm. Nullnorms were introduced in [4] as solutions of the Frank equation for uninorms.

Definition 2. (See [4].) The nullnorm F is a binary aggregation operator on [0, 1] that is commutative, associative and for which there is an element k in [0, 1] such that

$$F(x, 0) = x, \quad \text{for } x \leq k \quad \text{and} \quad F(x, 1) = x, \quad \text{for } x \geq k.$$