Available online at www.sciencedirect.com

ScienceDirect

Fuzzy Sets and Systems 291 (2016) 114–131

FUZZY sets and systems

www.elsevier.com/locate/fss

General aggregation operators based on a fuzzy equivalence relation in the context of approximate systems

Pavels Orlovs *, Svetlana Asmuss

Institute of Mathematics and Computer Science, University of Latvia, Raina blvd. 29, Riga, LV-1459, Latvia

Received 6 October 2014; received in revised form 20 May 2015; accepted 21 August 2015

Available online 28 August 2015

Abstract

Our paper deals with special constructions of general aggregation operators, which are based on a fuzzy equivalence relation and provide upper and lower approximations of the pointwise extension of an ordinary aggregation operator. We consider properties of these approximations and explore their role in the context of extensional fuzzy sets with respect to the corresponding equivalence relation. We consider also upper and lower approximations of a t-norm extension of an ordinary aggregation operator. Finally, we describe an approximate system, considering the lattice of all general aggregation operators and the lattice of all fuzzy equivalence relations.

Keywords: Aggregation operator; General aggregation operator; Fuzzy equivalence relation; Extensional fuzzy set; Upper and lower approximate operators; Approximate system

1. Introduction

In this paper we develop the concept of upper and lower general aggregation operators with respect to a fuzzy equivalence relation E. This concept was introduced and studied by the authors in [17,18]. In this paper we consider these operators as upper and lower approximations of a general aggregation operator and study their properties. In our previous works [15,16] we studied general aggregation operators based on a crisp equivalence relation instead of E. The idea was to aggregate fuzzy sets in accordance with classes of equivalence generated by this crisp equivalence relation. Taking into account that fuzzy equivalence relations represent the fuzzification of equivalence relations and extensional fuzzy subsets play the role of fuzzy equivalence classes, we consider the upper and lower general aggregation operators in the context of extensional fuzzy sets. It is important that the results of upper and lower general aggregation operators corresponding to fuzzy equivalence E are extensional with respect to E. In some cases while aggregating extensional fuzzy sets it could be necessary to obtain as a result an extensional fuzzy set as well, but an

* Corresponding author.
E-mail addresses: pavels.orlovs@gmail.com (P. Orlovs), svetlana.asmuss@lu.lv (S. Asmuss).

http://dx.doi.org/10.1016/j.fss.2015.08.015
0165-0114/

ordinary general aggregation does not ensure this property. We describe also upper and lower approximations of a t-norm extension of an ordinary aggregation operator.

The constructions of upper and lower general aggregation operators allow us to describe an approximate system (see e.g. [5,7,20,21]). Among the most important examples of approximate systems studied in [5,7,21] are approximate systems induced by fuzzy equivalence relations. These approximate systems are related to fuzzy rough sets, see e.g. [4].

The paper is organized as follows. In Section 2 we recall the definitions of an ordinary aggregation operator [1,3] and a general aggregation operator acting on fuzzy structures [22]. Then we give the definition of upper and lower general aggregation operators and show that these operators are general aggregation operators themselves. We illustrate by the numerical examples how these constructions operate. We show that upper and lower general aggregation operators preserve properties of fuzzy real numbers in the sense of B. Hutton [8], which are the special type of fuzzy sets. One subsection is devoted to the case of a crisp equivalence relation. Here we recall the construction of general aggregation operator, which aggregates fuzzy sets in accordance with classes of equivalence generated by crisp equivalence relation ρ.

In Section 3 we consider properties of upper and lower general aggregation operators with respect to extensional fuzzy sets. These operators are considered as aggregations which take values in the class of all extensional fuzzy sets, which could be treated as an advantage in some particular problems [13]. We show that upper and lower general aggregation operators are the best upper and lower approximations of a general aggregation operator respectively from such class of general aggregations.

In Section 4 we consider the constructions of upper and lower general aggregation operators based on a t-norm extension of an ordinary aggregation operator. We show that these operators also preserve the properties of inputs in the form of fuzzy real numbers.

Section 5 is devoted to approximate systems. We recall the definition of $\mathbb{M}$-approximate system [5,7,20,21], which provides an alternative view on the relations between fuzzy sets and rough sets. In the context of $\mathbb{M}$-approximate systems two lattices $\mathbb{L}$ and $\mathbb{M}$ play the fundamental role. We provide the construction $\mathbb{M}$-approximate system induced by fuzzy equivalence relation based on upper and lower general aggregation operators. This construction uses the lattice of all general aggregation operators and the lattice of all fuzzy equivalence relations.

2. Upper and lower general aggregation operators based on fuzzy equivalence relation

In this section first we will remind the definitions of an ordinary aggregation operator as well as a general aggregation operator, which acts on fuzzy structures. As the examples of widely used aggregation operators we can mention the arithmetic and geometric means, the minimum and maximum operators, t-norms and others.

2.1. General aggregation operators

Let us start with the classical notion of an aggregation operator (see, e.g., [1,3,6]).

Definition 2.1. A mapping $A : \bigcup_n [0,1]^n \to [0,1]$ is called an aggregation operator if and only if the following conditions hold:

$(A1)$ $A(0, \dots, 0) = 0$;
$(A2)$ $A(1, \dots, 1) = 1$;
$(A3)$ for all $n \in \mathbb{N}$ and for all $x_1, \dots, x_n, y_1, \dots, y_n \in [0,1]$:

$$x_1 \le y_1, \dots, x_n \le y_n \Longrightarrow A(x_1, \dots, x_n) \le A(y_1, \dots, y_n).$$

Conditions $(A1)$ and $(A2)$ are called the boundary conditions of A, but $(A3)$ means the monotonicity of A.

The general aggregation operator $\tilde{A}$ acting on $[0,1]^X$, where $[0,1]^X$ is the set of all fuzzy subsets of a set X, was introduced in 2003 by A. Takaci [22]. We denote an order on $[0,1]^X$ by $\preceq$. The least and the greatest elements of this order are denoted by $\tilde{0}$ and $\tilde{1}$, which are indicators of $\varnothing$ and X respectively, i.e.