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Fuzzy relational inequalities and equations, fuzzy quasi-orders, closures and openings of fuzzy sets ☆

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Abstract

The paper deals with systems of fuzzy relation equations known in the literature as eigen fuzzy set equations, as well as with systems of related fuzzy relation inequalities. Our main results are algorithms for computing the greatest and the smallest solutions to the considered systems of fuzzy relation inequalities, and algorithms for computing the greatest solutions to the considered systems of fuzzy relation equations. The systems are studied in the framework of complete residuated lattices as the structure of membership values, by means of fuzzy quasi-orders and closures and openings on a collection of fuzzy sets.
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1. Introduction

1.1. Origins

It is commonly known that throughout the history of fuzzy scientific research, fuzzy relations were early considered to be an efficient tool in various applications. For a contemporary general approach to fuzzy relations Bělohlávek's book [2] is among the best known, but there are also other general publications e.g., the books by Klir and Yuan [35], Turunen [49], see also Peeva and Kyosev, [39], Klawonn [34] and others. In the early period fuzzy relation equations and the corresponding systems were studied by Sanchez, with some applications in medical research (cf. [45–47], and later also [10,25]). Later on and presently, fuzzy relation equations and inequalities have been applied in fuzzy control, discrete dynamic systems, knowledge engineering, identification of fuzzy systems, prediction of fuzzy systems,

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decision-making, fuzzy information retrieval, fuzzy pattern recognition, image compression and reconstruction (cf., e.g., [14,17,19,20]).

Due to the aforementioned applications, the most frequently studied systems were the ones consisting of equations and/or inequalities containing the composition of a fuzzy relation and a fuzzy set on one side, while the other side contains another or the same fuzzy relation or fuzzy set. The unknown is either a fuzzy relation or a fuzzy set. These systems are often called *linear*. These were first studied in the above mentioned papers by Sanchez, who discussed linear systems over the Gödel structure.

Later, systems over more general structures of membership values were investigated, in particular those over complete residuated lattices (cf., e.g., [12,14,34,40,42,43]). More complex non-linear systems of fuzzy relation inequalities and equations, called weakly linear, have been recently introduced and studied in [28,29], see also [27] and [30,48].

Among many applications of fuzzy relations we point to fuzzy control. Essentially, it consists of an input, processing and output stages. It is usually rule based, and frequently fuzzy IF–THEN rules appear in this context, mostly in the processing stage. Practically, fuzzy rule bases use several antecedents that are combined using fuzzy operators or (fuzzifying if–then rules) fuzzy relations. The well known Mamdani approach to fuzzy controllers starts from a fuzzy relation which is deduced from control process, and which from an input value creates an output value using a suitable compositional rule of inference. In concrete applications of fuzzy control, it is frequent that output values are determined in advance by input values, and the problem is to find a fuzzy relation which performs such a transition. Obviously, such problems are formulated as above mentioned equations and inequalities, that is by linear systems. Another field of applications of analogous formulas is the so-called closedness under a fuzzy quasi-order given by Bodenhofer, De Cock and Kerre, in [9] (see also [31], and from the aspect of openings and closures on fuzzy sets, also [7]).

Some recent results arising from investigation of relation equations and inequalities in the framework of complete lattices are presented in [32,33].

1.2. Aim and structure of the paper

Motivated by the above mentioned applications (connected to fuzzy control, decision-making, image compression etc.) in this paper we deal with fuzzy relation inequalities of the form $u \circ R \leqslant u$ and $R \circ u \leqslant u$, and fuzzy relation equations of the form $u \circ R = u$ and $R \circ u = u$. By u we denote an unknown taking its values in the collection of all fuzzy subsets of a given set A, and by R a given binary fuzzy relation on A, both in the framework of lattice valued fuzzy structures over the same domain, the lattice being complete and residuated. The above equations are known in the literature as *eigen fuzzy sets equations*. We analyze systems of these inequalities and equations, we investigate their solvability, properties and structure of the set of their solutions. In particular, we provide algorithms for computing extremal solutions to these systems, and also, we highlight differences between systems of inequalities and equations. The main tools used in the paper are fuzzy quasi-orders (as well as fuzzy equivalences), residuals of fuzzy relations, and closures and openings on a lattice of fuzzy sets.

The paper is organized as follows. In Section 2 we give definitions of basic notions and notation concerning fuzzy sets and relations. In Section 3 we investigate fuzzy quasi-orders, their foresets and aftersets, and we identify the greatest fuzzy equivalence contained in a fuzzy quasi-order. On the related quotient set the former induces a fuzzy equality, and the latter a fuzzy order (with respect to this fuzzy equality). We also investigate other fuzzy equivalences contained in a fuzzy quasi-order and present some relevant properties of the corresponding quotient sets. In Section 4 we deal with systems of fuzzy relation inequalities of the form $u \circ R_i \leqslant u$ and $R_i \circ u \leqslant u$, with the unknown u and the given fuzzy relations R_i as above. Using the join of these fuzzy relations and then its fuzzy quasi-order (fuzzy equivalence) closure, we obtain equivalent single fuzzy relation inequality and single fuzzy relation equality. It also turns out that the solutions can be represented as linear combinations of foresets (aftersets, clases) of the corresponding fuzzy quasi-order and fuzzy equivalence. In particular, we provide algorithms for computing the greatest and the smallest solutions to the system of inequalities. Section 5 is devoted to systems of eigen fuzzy sets equations. We prove that the sets of solutions to the system is included in the corresponding set of solution to the single equation obtained by taking the join of fuzzy relations. We also describe the sets of solutions in terms of opening systems in the collection of all fuzzy sets on the domain. We present an algorithm which computes the greatest solution to the system of equations. Finally, in Section 6 we analyze and solve the problem of a characterization of collections of fuzzy subsets of a given set, which are sets of solutions of fuzzy relation equations, with the relation being a quasi-order.

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