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A discussion on fuzzy cardinality and quantification. Some applications in image processing

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Abstract

In this paper we discuss on some different representations of the cardinality of a fuzzy set and their use in fuzzy quantification. We have considered the widely employed sigma-count, fuzzy numbers, and gradual numbers. Gradual numbers assign numbers to values of a relevance scale, typically [0, 1]. Contrary to sigma-count and fuzzy numbers, they provide a precise representation of the cardinality of a fuzzy set. We illustrate our claims by calculating the cardinality of the fuzzy set of pixels that match a certain fuzzy color in an image. For that purpose we consider fuzzy color spaces previously defined by the authors, consisting of a collection of fuzzy sets providing a suitable, conceptual quantization with soft boundaries of crisp color spaces. Finally, we show the suitability of our approaches to fuzzy quantification for different applications in image processing. First, the calculation of histograms. Second, the definition of the notion of dominant fuzzy color, and the calculation of the degree to which we can say that a certain color is dominant in an image.

Keywords: Representation by levels; Fuzzy cardinalities; Color histograms; Dominant color; Gradual numbers; Fuzzy color spaces

1. Introduction

The usual framework for fuzzy quantification extends quantification based on the quantifiers ∃ and ∀, by allowing linguistic fuzzy quantifiers. In Zadeh's approach [55] two kinds of quantifiers are considered: **absolute quantifiers**, corresponding to fuzzy subsets of the non-negative integers of the form *around n* or *approximately between n and n′*, and **relative quantifiers**, corresponding to fuzzy subsets of the real interval [0, 1], representing imprecise percentages like *around a fraction q*, *approximately more than a fraction q*, etc. These quantifiers can be seen as fuzzy numbers defined as normalized, convex fuzzy sets defining restrictions on their respective domains. Other quantifiers have been also proposed following the theory of generalized quantifiers (TGQ) [3,27] which recognizes more than 30 types of quantifiers, but will not be discussed here.

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Fuzzy quantifiers are very useful in the setting of Computing with Words in order to generate linguistic statements about the number or percentage of objects that verify certain properties. Absolute quantifiers are employed in the so-called Type I sentences, with the form "Q of X are A", where Q is an absolute quantifier, X is a crisp set, and A is a fuzzy subset of X. Relative quantifiers are employed in Type II sentences, with the form "Q of D are A", where Q is a relative quantifier and D is a fuzzy subset of X.

Linguistically quantified sentences have been applied in a large amount of applications like data mining and data fusion, fuzzy control, fuzzy expert systems, decision-making, fuzzy queries in databases, linguistic summarization, fuzzy description logics, etc. We are interested in their application in the area of image processing, as a tool for helping to fill the semantic gap between the storage of images in computers and their description in terms of perceptual concepts employed by human beings. Many authors have employed fuzzy sets for representing semantic concepts like colors, texture features, and shapes. The notion of region is also perceptual, corresponding to a subset of pixels that is connected and homogeneous with respect to some semantic concept (or some combination of them), and several authors have argued that its most suitable representation is by means of fuzzy subsets of pixels, the so-called *fuzzy regions*. On this basis, other semantic concepts can be defined by means of quantified sentences. Some examples are:

- A fuzzy region is *large* when *At least a fraction q of the pixels in the image are in the region*, for some appropriate, user-defined value $q \in [0, 1]$. This is a type II sentence because the quantifier *At least a fraction q* is relative. The set D is the set of pixels in the image (a crisp set in this particular case), and the set A is the fuzzy region.
- A fuzzy region is *mostly red* when *At least a fraction q' of the pixels in the region are red*, for some appropriate $q' \in [0, 1]$. Here, the quantifier is *At least a fraction q'*, the set D is the fuzzy region, and the set A is the fuzzy set of pixels in the image that are *red*, where *red* is a *fuzzy color*, i.e., a fuzzy subset of crisp colors corresponding to our perception of red [41]. The set A can be easily obtained by computing the membership of each pixel's crisp color to the fuzzy set *red*.
- A fuzzy color is *dominant* in an image when *At least a fraction q'' of the pixels in the image are red*, for some appropriate $q'' \in [0, 1]$.
- A fuzzy region is *mostly in the center* of an image when *most of the pixels of the region are in the center of the image*. Here, the quantifier is *most*, the set D is the fuzzy region, and the set A is a fuzzy set of pixels with maximum membership in the geometrical center of the image and decreasing to the borders, that can be defined in different ways.

The fulfilment of quantified sentences is a matter of degree, and hence the semantic concepts that we can define using them are also fuzzy concepts. This is natural since quantifiers and the sets D and A are fuzzy sets. The accomplishment degree of quantified sentences can be calculated as the compatibility between the restriction defined by the quantifier, and a measure of the absolute cardinality of A (resp. relative cardinality of A with respect to D) for type I (resp. type II) sentences [55,16]. Hence, in order to develop methods for obtaining a reliable accomplishment degree, it is necessary to obtain good measurements of the absolute and relative cardinality of fuzzy sets.

Several authors have proposed ways to measure the cardinality of a fuzzy set, extending the classic one in different ways (see different compilations in [43,15,47]). Some of them have been employed for calculating the accomplishment of quantified sentences [55,51,16]. The most common approaches are the scalar cardinality and the fuzzy cardinality of a fuzzy set. The first approach claims that the cardinality of a fuzzy set is measured by means of a scalar value, either integer or real, whereas the second approach assumes the cardinality of a fuzzy set is just another fuzzy set over the non-negative integers. Among the latter, it is common to consider that the cardinality of a fuzzy set must be a fuzzy number, i.e., normalized and convex. However, this point has also been criticized [15]. Recently, an alternative called *gradual numbers* has been introduced by Dubois and Prade [21], that has been also employed in the evaluation of quantified sentences [31,37,38].

In this paper we discuss on some of the aforementioned representations of the cardinality of a fuzzy set and their use in fuzzy quantification. We show that fuzzy numbers are the best choice for representing restrictions like linguistic quantifiers, and their arithmetics is that of restrictions. On the other hand, gradual numbers are the best choice as measures of the cardinality of fuzzy sets. Hence, the evaluation of the accomplishment of quantified sentences is to be performed by calculating the compatibility between a fuzzy number and a gradual number.

We illustrate our claims by calculating the cardinality of the fuzzy set of pixels that match a certain fuzzy color in an image. For that purpose we consider fuzzy color spaces previously defined by the authors, consisting of a collection