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Single level constraint interval arithmetic ☆

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Abstract

In the present paper we propose a variant of constraint interval arithmetic that operates with a single parameter (level) in each interval operand of an expression. This results in a computationally simple interval arithmetic that has several desirable properties not shared in general by interval operations previously defined in the literature. The actualization and usefulness of this approach to fuzzy (interval) arithmetic is accomplished via alpha-levels. The evaluation of arithmetic expressions using the proposed single level constrained arithmetic is discussed in detail and we extend this theory to the fuzzy context.
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1. Introduction

Interval arithmetic has its foundations at least by the 1931 publication of R.C. Young's paper [33] on the algebra of many-valued quantities while functions of intervals, which is a branch of interval analysis, can be found in J.C. Burkill's 1924 paper [8], arguably the first interval analysis article. Even earlier, however, the spirit of interval computations was apparent in Archimedes' inner and outer approximation of the circumference of a circle. However, interval arithmetic as a complete and well-developed study first appeared with the 1956 publication [31] by the Polish mathematician M. Warmus. Independently in 1958, Japanese mathematician T. Sunaga published [28] on interval arithmetic. These are considered to be the first two publications on interval arithmetic as a subject unto itself. Still a third independent development of interval arithmetic is that of R.E. Moore's two 1959 papers [20,24]. While Warmus and Sunaga preceded Moore, neither Warmus nor Sunaga nor any of their collaborators went beyond the respective published papers. It was R.E. Moore and his collaborators who developed the whole area of interval analysis beginning with scientific computing (rounded interval arithmetic), differential equations, functional analysis, global

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optimization, and interval linear algebra all within the first ten years of Moore's first two papers. A historical and content development of interval arithmetic and analysis can be found in [11] and more extensively in [17].

We next mention the relevance of (single-level constrained) interval arithmetic to fuzzy (interval) arithmetic before we begin our development. Fuzzy arithmetic is interval arithmetic on α-*levels* [13]. Since fuzzy algebraic operations of addition, subtraction, multiplication, and division by non-zero values are continuous functions, Zadeh's extension principle applied to these operations yields interval arithmetic on α-*levels* since the α-*levels* of fuzzy numbers are intervals. This relationship, between fuzzy intervals and real-valued intervals along with interval/fuzzy arithmetic and fuzzy/interval analysis, is fully developed in [17]. Thus, what we present has intimate relationship to fuzzy arithmetic and analysis which we do not always make explicit for the sake of brevity and since it has already been articulated. However in several cases we will present the fuzzy arithmetic operations derived from the proposed interval operations.

Moore's development of interval arithmetic is accepted as the standard approach to interval arithmetic which we will call standard interval arithmetic (SIA) and it is the approach to interval arithmetic in common use. Moore's primary concern in developing interval arithmetic was two-fold as we understand it.

(1) In all computations including (or most especially) those performed on a digital computer should *never lose any possible value*. This was the thrust of his first two 1959 papers. This thrust came out of his work as a computer scientist. As one of the first computational numerical analysts, his concerns were in roundoff errors from computational methods being implemented on digital computers especially in light of the work of Von Neumann and Goldstein's paper [30] and Wilkinson's work (see [32] – this 1971 publication summarizes his earlier work).
(2) Compute bounds on interval calculations not only so that one never loses possible values, but one computes these quickly.

Let us see how these two concerns impacted interval arithmetic since it is how to deal with ensuing limitations of Moore's standard interval arithmetic that this study addresses. The basic standard interval arithmetic definitions are fully developed in [21]. As is well known, the definitions for interval arithmetic are ($a \le b, c \le d$):

(1) $[a, b] + [c, d] = [a + c, b + d]$
(2) $[a, b] - [c, d] = [a - d, b - c]$
(3) $[a, b] \times [c, d] = [\min\{ac, ad, bc, bd\}, \max\{ac, ad, bc, bd\}]$
(4) For $0 \notin [c, d]$, $[a, b] \div [c, d] = [\min\{\frac{a}{c}, \frac{a}{d}, \frac{b}{c}, \frac{b}{d}\}, \max\{\frac{a}{c}, \frac{a}{d}, \frac{b}{c}, \frac{b}{d}\}]$

Definition 1. The interval arithmetic associated with 1–4 above we call **standard interval arithmetic (SIA)**.

There are various properties associated with SIA which are different from those of real numbers and that of the constrained interval arithmetic (CIA) defined subsequently. In particular, interval arithmetic derived from the definitions above, SIA, is subdistributive. Thus, from [22] we have for intervals $[x]$, $[y]$, and $[z]$:

(1) $[x] + ([y] + [z]) = ([x] + [y]) + [z]$ – the associative law for addition
(2) $[x] \cdot ([y] \cdot [z]) = ([x] \cdot [y]) \cdot [z]$ – the associative law for multiplication
(3) $[x] + [y] = [y] + [x]$ – the commutative law for addition
(4) $[x] \cdot [y] = [y] \cdot [x]$ – the commutative law for multiplication
(5) $[0, 0] + [x] = [x] + [0, 0] = [x]$ – additive identity
(6) $[1, 1] \cdot [x] = [x] \cdot [1, 1] = [x]$ – multiplicative identity
(7) $[x] \cdot ([y] + [z]) \subseteq [x] \cdot [y] + [x] \cdot [z]$ – the subdistributive property

The full study and computer implementation of SIA was completed by [15] whose rounded (standard) interval arithmetic resulted in an algebraic structure they called a *ringoid*. This was incorporated into an IBM product, later in PASCAL XSC and C++ XSC. The successor of these efforts is a MATLAB implementation called INTLAB.

The power of SIA is that it is simple to apply. Its complexity is at most four times that of real-valued arithmetic (less on modern computers). However, SIA leads to overestimations in general because it takes every instantiation

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