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Interval and fuzzy Average Internal Rate of Return for investment appraisal ☆

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Abstract

In investment appraisal, uncertainty can be managed through intervals or fuzzy numbers because the arithmetical properties and the extension principle are well established and can be successfully applied in a rigorous way. We apply interval and fuzzy numbers to the Average Internal Rate of Return (AIRR), recently introduced for overcoming the problems of the traditional Internal Rate of Return (IRR). In the setting of interval and fuzzy arithmetic, we establish relations between the interim capitals invested, the profits and the cash flows, which are the ingredients of the AIRR and shed lights on the different ways uncertainty propagates depending on which variable is known and which one is derived. The relations between fuzzy AIRR and fuzzy Net Present Value are also investigated.

1. Introduction

Investment appraisal is a central issue in the literature and real-life applications. While the Net Present Value (NPV) is widespread as a measure of value created, it is most common in real-life applications to require a rate of return, for a relative measure is more intuitive than an absolute measure. The Internal Rate of Return (IRR) is the standard rate of return used by decision maker. However, it has some well-known shortcomings: (i) it may not exist, (ii) multiple IRRs may exist, (iii) it may be incompatible with the NPV, (iv) it cannot be employed with variable costs of capital. An alternative rate of return overcoming these flaws is the Average Internal Rate of Return (AIRR), proposed in [30], which is defined as the ratio of the aggregate return generated by the project to the aggregate capital invested in the project.

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In this paper we introduce uncertainty in investment appraisal when economic profitability is measured with the AIRR criterion (a preliminary approach is in [19]). The managing of uncertainty is developed by modeling variables as intervals and fuzzy numbers (Buckley in [6] introduces the possible contribution of fuzzy mathematics in finance, as in [18]). This approach is not new in literature and some authors introduce alternative versions of classical criteria achieving interesting results. In all cases, we will consider that the data are affected by uncertainty or imprecision in the epistemic sense, i.e. we will consider intervals and fuzzy quantities as disjunctive sets (see the discussion in [14]).

Chiu and Park propose in [9] a decision model in which the uncertain cash flows and discounted rates are specified as fuzzy numbers. They study the difference between the present worth and the approximated present worth formula and observe that the range of periodic cash flows is not a key factor, while a larger range of discount rates increases the distance between the two formulas. The fuzzy project selection is performed by applying different dominance rules.

In a second paper [10], the same authors extend the case of capital budgeting problems with exact numbers to the case of cash flows information modeled with triangular fuzzy numbers. The selection of fuzzy projects is handled by a branch and bound procedure.

The studies in [21] go into the direction of studying the possible ways to model uncertainty in economics; in particular, they show that robustness of capital budgeting techniques depends on the statistical estimation of cash amounts and interest rates, exhibiting often hard difficulties that can be captured by fuzzy numbers. In [37] methods for integrating probability and possibility distributions are discussed and a computer simulation used for an investment project risk assessment shows that it is possible to model some parameters with fuzzy numbers and others with a probability distribution.

Kuchta in [25] proposes generalized fuzzy equivalents in order to incorporate uncertainty into the most commonly used techniques for capital budgeting. These fuzzy equivalents allow to evaluate projects whose cash flows and/or duration are not known precisely, but given in the form of fuzzy numbers. In [17] the authors follow the Kuchta approach with particular attention to the parametric representation of fuzzy numbers. In a second paper (details in [26]) Kuchta assumes a fuzzy Net Present Value for some projects and shows that the common realization of them may allow savings in the resource utilization.

In [2] the concept of possibilistic mean and variance is extended to adaptive fuzzy numbers and applied to the computation of the fuzzy net present value of future cash flows, while in [4] the uniqueness of a fuzzy IRR is proved under suitable hypothesis.

Also, in [20], the underlying hypothesis is that fuzzy variables can reflect uncertainty of investment outlays, annual net cash flows and investment capital; a new mean-variance model based on credibility measure is proposed for optimal capital allocation.

The potentialities of the fuzzy approach are highlighted also in [38]: fewer assumptions about the data distribution and market behavior are enough to describe fluctuations that go beyond the probability model. A measure of the risk associated with each investment opportunity and an estimate of the projects' robustness towards market uncertainty is then derived.

A fuzzy logic system in [41] is adopted to extend the classical discounted cash flow model in order to take into account the uncertain information intrinsic in the value of a company's financial assets. The paper [36] establishes that the definition of IRR is ambiguous for fuzzy data and leads to an IRR which is a fuzzy number of type 2.

In [27] a fuzzy binomial approach for project valuation under uncertainty is proposed and a method to compute the mean value of a project's fuzzy NPV is provided. Using the credibilistic expected value, the authors in [43] solve the problems of project valuation and portfolio selection under the assumption that the investment capitals and the net cash flows of the projects are fuzzy variables.

In two recent books, Carlsson and Fullér [7] fully describe the possibility theory applied to real life decisions and Dymowa in [11] explores financial applications of soft computing and in particular she presents the generalized method for multiple criteria evaluation of investments in the fuzzy setting.

The main goal of this paper is to investigate the AIRR concept and criterion in the interval and the fuzzy context, and to analyze how theoretical relations and equations between variables in the scalar setting can be extended to the interval or fuzzy context.

The existence and the validity of equations between scalar variables produce constraints to the possible values, when variables are extended by introducing uncertainty with intervals and fuzzy numbers. We show that the extended arithmetic operations $+, \oplus, -, \ominus, \cdot, \odot, /, \oslash$ in the AIRR application, can be used in conjunction with the constrained extension principle to express interval and fuzzy forms of the scalar relations between the variables. In this way we