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Robust control design for nonlinear stochastic partial differential systems with Poisson noise: Fuzzy implementation

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Abstract

This study addresses the robust H_∞ control design problem for nonlinear stochastic partial differential systems (NSPDSs) with Poisson noise under the environment of random external disturbance in the spatio-temporal domain. For NSPDSs with Poisson noise, the robust H_∞ control design needs to solve a complex Hamilton–Jacobi integral inequality (HJII) for robust control despite random external disturbance. In general, it is very difficult to solve the nonlinear partial differential HJII. In order to simplify the design procedure, a fuzzy stochastic partial differential system is proposed to approximate the NSPDS based on fuzzy interpolation approach. Then a fuzzy stochastic spatial state space model is developed to represent the fuzzy stochastic partial differential system via the semi-discretization finite difference scheme and the Kronecker product. Based on this model the robust H_∞ control design is proposed to achieve the robust control of NSPDSs via solving linear matrix inequalities (LMIs) instead of an HJII. The proposed robust fuzzy H_∞ controller has an efficient ability to attenuate the effect of spatio-temporal external disturbance on the controlled output of NSPDSs from the area energy point of view. Finally, a robust H_∞ control of the nervous system is given to confirm the control performance of the proposed robust control design method for NSPDSs with Poisson noise.

Keywords: Fuzzy approach; Robust H_∞ controller; LMI; Nonlinear stochastic partial differential system; Spatio-temporal domain

1. Introduction

The H_∞ control problem is to design a controller to control the system to guarantee the $\mathcal{L}_2$ gain (from the external disturbance to the controlled output) to be of less than a prescribed level [1–3]. In recent years, the stochastic H_∞ control problem with system models expressed by Ito-type stochastic differential systems has become a popular research topic and has gained extensive attentions; see [1–3] and the references therein. In [1], a bounded real lemma

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was presented for linear continuous-time stochastic systems. In [2] and [3], the linear and nonlinear stochastic H_∞ control problems have been discussed.

Recently, there have been many works concerning the study of partial differential systems (PDSs) [4–13]. Many fields have been concerned with deterministic partial differential systems, for example, chemical reactor systems [4], electromagnetic transmission systems [5], image processing systems [6], manufacturing flow systems [7], neurophysiological systems, biodynamic systems, heat transfer systems, fluid dynamic systems, population dynamic systems, elastic wave transmission systems, flexible dynamic systems [8], etc. Since the most of phenomena have some uncertainties due to the existence of different stochastic fluctuations, the more accurate representation of the behaviors should be modeled by stochastic partial differential system (SPDS) [4,9–11,13]. In this study, we focus on the robust control problem of nonlinear stochastic partial differential systems (NSPDSs), which is an important topic in the system control design.

On the other hand, in many realistic situations, the noise for a dynamical system may be Wiener or Poisson process. Poisson process has the random discontinuity in the state dynamics [14,15], whereas Wiener process is continuous fluctuation or diffusion in the system. In [16], the Gaussian white noise current which is used to represent an approximation to Poisson trains of excitatory and inhibitory potentials in the Hodgkin–Huxley equations is modeled in the neural stochastic systems. As mentioned above, the behavior of systems with Poisson noises is quite different from that of systems with Wiener noises, i.e., Poisson noises cause random discontinuities of the system state, whereas Wiener noises cause continuous perturbations of the system state. This feature of Poisson noises makes the control for systems with Poisson noises harder to deal with than the control for systems with Wiener noises. So this feature of Poisson noises interests us to investigate the robust control design for stochastic partial differential systems with Poisson noise.

In this study, the general robust control theory for NSPDSs with Poisson noise is studied from the spatio-temporal H_∞ disturbance attenuation point of view. Then the H_∞ nonlinear partial differential controller is designed for NSPDSs based on the general robust control theory. For the robust control of NSPDSs with Poisson noise, the robust H_∞ control design needs to solve a complex Hamilton Jacobi integral inequality (HJII) which is with an integration of the Hamilton Jacobi inequality (HJI) over the space domain U, whereas the robust H_∞ control design for nonlinear stochastic ordinary differential systems (NSODSs) only needs to solve a corresponding HJI. In general, it is very difficult to solve a corresponding HJII for the robust H_∞ nonlinear controller. In this situation, fuzzy interpolation method and finite difference method are employed to simplify the design procedure of the robust H_∞ controller for NSPDSs. Recently, the fuzzy approach [17–24] has been widely used in many fields.

The stabilization design problem of nonlinear partial differential systems (NPDSs) has been discussed based on Galerkin's method [11]. But the infinite dimensional truncation of Galerkin's method is too complex. To avoid this design difficulty, we shall use the fuzzy interpolation method and finite difference method to simplify the design procedure. In this study, based on the fuzzy interpolation, the finite difference scheme and the Kronecker product, the states of all finite difference grids in the spatio-domain are formulated as a spatial state vector so that a T–S fuzzy spatial state space system can represent an NSPDS with some approximation errors. In this situation, the spatio-temporal H_∞ control performance can be transformed to the equivalent temporal H_∞ control performance so that the implementable fuzzy H_∞ spatial controller can be designed to robustly control the state variables of NSPDSs with Poisson noise. In this situation, a set of LMIs are used to replace the nonlinear partial differential HJII to simplify the design procedure of robust control problem of NSPDSs through the use of fuzzy interpolation and finite difference schemes. Finally, a robust H_∞ control example for NSPDSs is given to confirm the H_∞ control performance of the proposed robust control for NSPDSs with Poisson noise. The contributions of the proposed method include: (i) The spatio-temporal H_∞ control performance is employed to treat the robust control problem for NSPDSs with Poisson noise under the environment of random external disturbance $v(x,t)$ in the spatio-temporal domain. (ii) Based on HJII, the more general spatio-temporal H_∞ control design theory is developed for NSPDSs with Poisson noise. (iii) The fuzzy interpolation scheme, the finite difference method and the Kronecker product technique are combined to transform the NSPDS with Poisson noise to an equivalent fuzzy spatial state space system so that an implementable H_∞ fuzzy spatial controller could be easily designed for the NSPDS with Poisson noise via solving a set of LMIs through the help of Matlab LMI toolbox.

$\|x\|$: the Euclidean norm of a vector x.

$\|A\| := \sup_{x\neq 0} \frac{\|Ax\|}{\|x\|} = \sqrt{\lambda_{\max}(A^T A)} = \sigma_{\max}(A)$ for a matrix A, where $\sigma_{\max}(A)$ is the maximum singular value of A.