

# $H_\infty$ fuzzy control for nonlinear time-delay singular Markovian jump systems with partly unknown transition rates

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Received 17 February 2013; received in revised form 28 August 2013; accepted 27 December 2013

Available online 2 January 2014

## Abstract

This paper investigates the problem of  $H_\infty$  fuzzy control for a class of nonlinear time-delay singular Markovian jump systems with partly unknown transition rates. This class of systems under consideration is described by Takagi–Sugeno (T–S) fuzzy model. The goal of the paper is to design fuzzy state-feedback controllers such that systems with partly unknown transition rates are not only regular, impulse-free and stochastically stable, but also satisfy a prescribed  $H_\infty$  performance for all delays no larger than a given upper bound in terms of linear matrix inequalities. And systems with completely known transition rates can be viewed as a special case of the one we consider here. Finally numerical examples are given to illustrate the merit and usability of the approach proposed in this paper.

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**Keywords:** Fuzzy control;  $H_\infty$  control; Stochastic admissibility; Singular Markovian jump systems; Takagi–Sugeno fuzzy model

## 1. Introduction

A lot of attention has been paid to the investigation of Markovian jump systems (MJSs), which are a special kind of hybrid systems, because they have the advantage of better representing physical systems with random changes in their structures and parameters, and have successful applications in control and communication fields. Many important issues have been studied for this kind of systems, such as stability analysis, stabilization,  $H_\infty$  and  $H_2$  control [1–4]. Singular systems, which are also referred to as generalized state space systems, descriptor systems, implicit systems, or differential-algebraic systems, have attracted many researchers due to the fact that singular systems have extensive applications in electrical circuits, power systems economics and so on [5,6]. When singular systems experience abrupt changes in their structures, it is natural to model them as singular Markovian jump systems (SMJSs) [7–9]. Time delays are always sources of poor stability or performance of a system. So study of stability criteria and performance for time-delay SMJSs are of theoretical and practical importance [10–12].

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The past decade has witnessed rapidly growing interest in fuzzy control of nonlinear systems. In particular, the so-called T–S fuzzy model has been employed for control design of nonlinear systems [13–15]. Based on the T–S model, the fruitful linear system theory can be applied to analysis and controller synthesis of nonlinear systems. In recent years, this fuzzy-model-based technique has been used to deal with nonlinear time-delay systems and nonlinear MJFSs. For example, mode-independent and mode-dependent fuzzy control designs of Markovian jump fuzzy systems (MJFSs) have been presented in [16–19]. In fact, in many cases, transition rates of MJFSs are not exactly known. So it is necessary to further consider MJFSs with partial information on transition rates [20,21]. To the best of our knowledge, the problem of stochastic admissibility for time-delay singular Markovian jump fuzzy systems (SMJFSs) has not been fully investigated, not to deal with the problem of the stochastic admissibility for this kind of systems with partly unknown transition rates. Moreover, there has been a considerable process in the stability analysis and  $H_\infty$  controller synthesis for time-delay fuzzy singular systems [15,22,23] and time-delay linear SMJFSs [10–12]. However, corresponding problems for time-delay SMJFSs with completely known (partly unknown) transition rates remain to be investigated.

In this paper, we are concerned with the  $H_\infty$  fuzzy control problem for a class of nonlinear time-delay singular Markovian jump systems with partly unknown transition rates, which can be represented by the T–S fuzzy model. Our aim is to design fuzzy state-feedback controllers for the time-delay SMJFS with partly unknown transition rates, which takes the time-delay SMJFS with completely known transition rates as its special case, such that the closed-loop system is stochastically admissible (regular, impulse-free and stochastically stable) with a prescribed performance of disturbance attenuation. Firstly, based on Lyapunov–Krasovskii function, a delay-dependent admissibility criterion is presented by strict linear matrix inequalities (LMIs). Then we design a mode-dependent  $H_\infty$  fuzzy controller. With this fuzzy controller, the closed-loop system is stochastically admissible with  $H_\infty$  performance  $\gamma$ . Finally numerical examples are given to demonstrate the effectiveness of the approach proposed here.

**Notation.** Throughout this paper,  $\lambda_{\min}(A)$  means the minimal eigenvalue of the real square matrix  $A$ .  $L_2[0, \infty)$  stands for the space of square integrable functions on  $[0, \infty)$ .  $(\Omega, \mathcal{F}, \mathcal{P})$  is a probability space,  $\Omega$  is the sample space,  $\mathcal{F}$  is the  $\sigma$ -algebra of subsets and  $\mathcal{P}$  is the probability measure on  $\mathcal{F}$ .  $\mathcal{E}\{\cdot\}$  denotes the expectation operator with respect to some probability measure  $\mathcal{P}$ .  $\|\cdot\|$  stands for the Euclidean norm for a vector.  $C_{n,d} = C([-d, 0], \mathbb{R}^n)$  denotes Banach space of continuous vector functions mapping interval  $[-d, 0]$  into  $\mathbb{R}^n$  with norm  $\|\phi(t)\|_d = \sup_{-d \leq s \leq 0} \|\phi(s)\|$ . In addition, the symbol ‘\*’ denotes the term which can be inferred by symmetry and  $\text{diag}\{\cdot\cdot\cdot\}$  stands for a block-diagonal matrix.

## 2. Basic definitions and lemmas

Fix a probability space  $(\Omega, \mathcal{F}, \mathcal{P})$ , and consider a time-delay SMJFS, its  $i$ th fuzzy rule is given by

**R<sub>i</sub>:** If  $\xi_1(t)$  is  $M_{i1}$ , and  $\xi_2(t)$  is  $M_{i2}, \dots$ , and  $\xi_l(t)$  is  $M_{il}$  Then

$$\begin{cases} E\dot{x}(t) = A_i(r_t)x(t) + A_{d,i}(r_t)x(t-d) + B_i(r_t)u(t) + B_{\omega,i}(r_t)\omega(t) \\ z(t) = C_i(r_t)x(t) + C_{d,i}(r_t)x(t-d) + D_i(r_t)u(t) + C_{\omega,i}(r_t)\omega(t) \\ x(t) = \phi(t), \quad t \in [-\bar{d}, 0], \quad i \in \mathcal{T} \triangleq \{1, 2, \dots, k\} \end{cases} \tag{1}$$

where  $x(t) \in \mathbb{R}^n$  is the state vector,  $u(t) \in \mathbb{R}^m$  is the control input,  $\omega(t) \in \mathbb{R}^q$  is a disturbance which belongs to  $L_2[0, \infty)$ ,  $z(t) \in \mathbb{R}^p$  is the controlled output.  $d$  is an unknown constant delay satisfying  $d \in [0, \bar{d}]$ ,  $\phi(t) \in C_{n,\bar{d}}$  is a compatible vector valued initial function.  $k$  is the number of If–Then rules,  $M_{ij}$  ( $i \in \mathcal{T}, j = 1, 2, \dots, l$ ) are fuzzy sets,  $\xi_1(t), \xi_2(t), \dots, \xi_l(t)$  are premise variables.  $E \in \mathbb{R}^{n \times n}$  is a singular matrix with  $\text{rank } E = r < n$ ,  $A_i(r_t), A_{d,i}(r_t), B_i(r_t), B_{\omega,i}(r_t), C_i(r_t), C_{d,i}(r_t), D_i(r_t)$  and  $C_{\omega,i}(r_t)$  are known constant matrices with appropriate dimensions.  $\{r_t, t \geq 0\}$  is a continuous-time Markov process with right continuous trajectories taking values in a finite set given by  $S = \{1, 2, \dots, N\}$  with the transition rates matrix (TRM)  $\Pi \triangleq \{\pi_{pq}\}$  given by

$$\Pr\{r_{t+h} = q \mid r_t = p\} = \begin{cases} \pi_{pq}h + o(h), & p \neq q \\ 1 + \pi_{pp}h + o(h), & p = q \end{cases}$$

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