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Fuzzy control of continuous-time recurrent fuzzy systems

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Abstract

This article presents a synthesis method for stabilization of equilibria in continuous-time recurrent fuzzy system. It is first shown how to obtain a hybrid system representation from the given linguistic differential equations defining the recurrent fuzzy system. Based on this, the controller synthesis algorithm presented is then a two step procedure. By means of the synthesis step, a fuzzy controller is obtained by solving a single optimization problem, which guarantees that no chattering effects will occur in the controlled recurrent fuzzy system. Stability of the equilibrium is then proven by a verification procedure utilizing sum of squares methods. A performance optimization by means of additional polynomial controllers is discussed. The main advantage of our method relies in the computational efficiency of the synthesis problem as well as in the linguistic interpretability of the resulting controller. In addition, chattering effects are proven not to occur.

Keywords: Recurrent fuzzy systems; Hybrid systems; Fuzzy control; Sum-of-squares

1. Introduction

Modeling system dynamics based on expert knowledge has become increasingly popular ever since the introduction of fuzzy logic. Besides static Mamdani fuzzy systems [1], a vast number of dynamic fuzzy systems have emerged during the last decades. The well-known Takagi–Sugeno type fuzzy systems [2], fuzzy differential equations [3] or fuzzy dynamical systems [4] may serve as examples (see [5] for a survey). In addition, a concise theory of recurrent fuzzy systems has been developed during the last decade, which in contrast to other fuzzy systems with dynamics allows for a linguistic description of the system dynamics, whereas state, input and derivative take on crisp values. Being first described in the literature in [6] and [7], system properties were analyzed mathematically in [8] for recurrent fuzzy systems in discrete time and [9] in the continuous time case. In [10], the modeling of system dynamics was then studied in depth, driven by an interest in practical applicability of recurrent fuzzy systems for system representation. In addition, applications for fault diagnosis purposes appear promising [11]. Recent works [12,13] were for the first time concerned with the synthesis of controllers for continuous-time recurrent fuzzy systems.

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Besides the approach of utilizing fuzzy logic in order to model system dynamics, hybrid and switched systems [14–16] have received much attention over the last years. They naturally appear in processes having continuous as well as discrete states, but their use can also be motivated by the fact that many nonlinear systems can be approximated by locally defined linear or affine systems [17]. Thus, numerous articles have been devoted to the stability analysis of switched and hybrid systems, e.g. [18–20].

In this article, we approach continuous-time recurrent fuzzy systems from a hybrid system's view point and present a new synthesis procedure for (static) fuzzy controllers. As will be shown, one major benefit of this approach is the continuity of the dynamic function of the closed loop system by means of the proposed controller. This property is particularly important, since in switched and hybrid systems, chattering effects may occur [15]. The synthesis method mainly consists of two steps: First, sufficient conditions for asymptotic stability of an equilibrium are derived, whereas the resulting feasibility problem is solved approximately. In a second verification step, a Lyapunov function is calculated proving stability of the closed-loop system. Furthermore, this basic idea is extended towards linguistic interpretability of the resulting fuzzy controller and performance optimization of the controller operating around the origin.

This multi-step procedure is in contrast to the previous work [13], in which a piecewise polynomial controller and a Lyapunov function were obtained simultaneously by solution of a single sums of squares problem. The drawback of this approach is mainly its limitation due to the complexity of the resulting optimization problem even for recurrent fuzzy systems of low dimension. The complexity is reduced significantly by means of the procedure presented in this article, which computes a stabilizing controller without the use of the sums of squares framework. Instead, only a system of (in)equalities has to be solved, which is linear in the SISO-case. The verification of the stability of the controlled system can then be shown by means of a sums of squares problem, in which only the Lyapunov function is unknown and is thus easier to solve than in the case of controller and Lyapunov function being unknown.

The remainder is organized as follows: In Section 2, we review basic definitions of continuous-time recurrent fuzzy systems and introduce some notation. Section 3 then introduces a motivating small scale example, which is considered throughout the article for illustration of the theory. The main results are then presented in Section 4 which describes the synthesis method in detail. In addition, aspects on linguistic interpretability and hybrid phenomena are discussed. A verification procedure based on sum of squares follows in Section 5, whereas an approach for performance optimization around the equilibrium is discussed in Section 6. Concluding remarks are given in Section 7.

2. Preliminaries

2.1. Recurrent fuzzy systems

In the following, we briefly review the formal definition of continuous-time recurrent fuzzy systems (CTRFS) according to [9], to which we refer for additional details. CTRFS are defined in the input state space $\mathcal{Z} = \mathcal{X} \times \mathcal{U} \subseteq \mathbb{R}^{n+m}$ with states $\mathbf{x} \in [\mathbf{x}_{\min}, \mathbf{x}_{\max}]$, inputs $\mathbf{u} \in [\mathbf{u}_{\min}, \mathbf{u}_{\max}]$ and $\mathbf{z} = [\mathbf{x}^T, \mathbf{u}^T]^T$. The dynamics of the system are defined linguistically by a finite number of linguistic differential equations, which are rules of the form

$$\begin{aligned} &\text{If } x_1 = L_{j_1}^{x_1}, \text{ and } \dots, \text{ and } x_n = L_{j_n}^{x_n}, \\ &\text{and } u_1 = L_{q_1}^{u_1}, \text{ and } \dots, \text{ and } u_m = L_{q_m}^{u_m}, \\ &\text{then } \dot{x}_1 = L_{w_1(\mathbf{j},\mathbf{q})}^{\dot{x}_1}, \text{ and } \dots, \text{ and } \dot{x}_n = L_{w_n(\mathbf{j},\mathbf{q})}^{\dot{x}_n}. \end{aligned} \tag{1}$$

The complete set of rules is called the *rule base*. By $L_{j_i}^{x_i}$ and $L_{q_i}^{u_i}$, linguistic values of the state- and input variables in the i-th dimension of $\mathcal{X}$ and $\mathcal{U}$ are denoted, describing them in qualitative terms like *low*, *neutral* or *hot*. Similarly, state derivatives $\dot{x}_i$ are described by linguistic values $L_{w_i(\mathbf{j},\mathbf{q})}^{\dot{x}_i}$. In order to quantify the system dynamics, each linguistic value $L_{j_i}^{x_i}$ ($L_{q_i}^{u_i}$) is associated with a crisp value $s_{j_i}^{x_i}$ ($s_{q_i}^{u_i}$), called *core position*. For the description of the state derivatives, linguistic values $L_{w_i}^{\dot{x}_i}$ are linked with *core position derivatives* $s_{w_i}^{\dot{x}_i}$. For simplicity, a vectorial notation $\mathbf{L}_{\mathbf{j}}^{\mathbf{x}} = [L_{j_1}^{x_1}, \dots, L_{j_n}^{x_n}]^T$, $\mathbf{L}_{\mathbf{q}}^{\mathbf{u}} = [L_{q_1}^{u_1}, \dots, L_{q_m}^{u_m}]^T$ and $\mathbf{L}_{\mathbf{w}(\mathbf{j},\mathbf{q})}^{\dot{\mathbf{x}}} = [L_{w_1(\mathbf{j},\mathbf{q})}^{\dot{x}_1}, \dots, L_{w_n(\mathbf{j},\mathbf{q})}^{\dot{x}_n}]^T$ for linguistic values is used. Similarly, $\mathbf{s}_{\mathbf{j}}^{\mathbf{x}}$, $\mathbf{s}_{\mathbf{q}}^{\mathbf{u}}$, $\mathbf{s}_{\mathbf{w}}^{\dot{\mathbf{x}}}$ denote core position vectors and their derivatives. Thus, a more compact representation of (1) is obtained: