Available online at www.sciencedirect.com

ScienceDirect

Fuzzy Sets and Systems 281 (2015) 73–87

FUZZY sets and systems

www.elsevier.com/locate/fss

Evaluative linguistic expressions vs. fuzzy categories ☆

Vilém Novák [a,b,*]

[a] *University of Ostrava, Institute for Research and Applications of Fuzzy Modeling, NSC IT4Innovations, 30. dubna 22, 701 03 Ostrava 1, Czech Republic*

[b] *Institute of Mathematics and CS, University of Latvia, LV-1459, Riga, Latvia*

Received 13 October 2014; received in revised form 17 August 2015; accepted 25 August 2015

Available online 3 September 2015

Abstract

In this paper, we discuss the distinction between categories characterized by verbal labels taken from a fuzzy rating scale and special class of linguistic expressions, called evaluative. The latter form a general class of expressions that includes gradable and evaluative adjectives and their hedges. First, we will provide a brief linguistic analysis of them. Then we outline basic principles for construction of the mathematical model of semantics of evaluative expressions. In Section 3 we will analyze the concepts of rating scale with verbal labels (fuzzy rating scale), their semantics and demonstrate that the latter cannot be identified with the semantics of evaluative expressions. Finally, we suggest a method for construction of fuzzy rating scales on the basis of evaluative linguistic expressions. The main goal of this paper is to demonstrate the power of fuzzy set theory and to show that it is capable of providing tools for advanced modeling of linguistic semantics.

Keywords: Evaluative linguistic expression; Rating scale; Verbal label; Fuzzy natural logic; Linguistic hedges; Fundamental evaluative trichotomy

1. Introduction

L.A. Zadeh, the founder of fuzzy set theory, demonstrated in many of his papers, for example [52,56–59], that his theory makes it possible to develop a new mathematical model of semantics of some expressions of natural language and to apply it in solution of various practical problems (see, e.g., [3,55] and elsewhere). One of the most successful applications is fuzzy control that was initiated by Zadeh in [54] but first elaborated to a working algorithm by Mamdani and Assilian [26] and then further developed by many other authors. The main reason for its success is probably the possibility to realize control by transforming operator's (expert's) knowledge formulated in natural language into an algorithm.

☆ The support of the ESF project 2013/0024/1DP/1.1.1.2.0/13/APIA/VIAA/045 is kindly announced. Additional support is given by the European Regional Development Fund in the IT4Innovations Centre of Excellence project (CZ.1.05/1.1.00/02.0070).

* Correspondence to: University of Ostrava, Institute for Research and Applications of Fuzzy Modeling, NSC IT4Innovations, 30. dubna 22, 701 03 Ostrava 1, Czech Republic.

E-mail address: Vilem.Novak@osu.cz.

http://dx.doi.org/10.1016/j.fss.2015.08.022

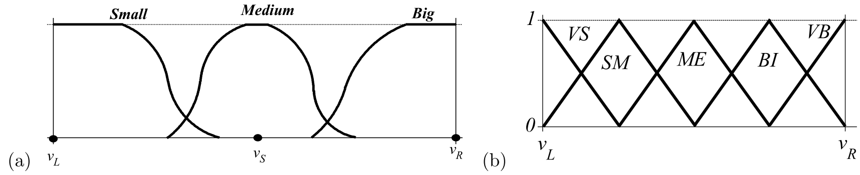

Fig. 1. (a) Fuzzy sets modeling the meaning of words forming the fundamental linguistic trichotomy (Z-, Π- and S-functions). (b) Triangular fuzzy sets applied in fuzzy control and many other applications of fuzzy set theory. The shorts are as follows: VS—very small, SM—small, Me—medium, BI—big, VB—very big.

The fundamental role in these applications is played by expressions such as "small, very small, medium, big" etc. Zadeh suggested to model their meaning using special fuzzy sets defined on an ordered universe. Moreover, he also came with the idea that semantics of the, so called, *linguistic hedges* can be modeled by special operations on the corresponding membership functions. This made it possible to compute semantics of more complex expressions. For example, if $A_{small}(x)$ is a membership function of the fuzzy set modeling the meaning of "small" then the meaning of "very small" is modeled by the fuzzy set $A^2_{small}(x)$ and that of "more or less small" by $A^{0.5}_{small}(x)$. Later, he extended the repertoire of possible functions to more linguistic hedges so that the meaning of other complex expressions could also be modeled (see, e.g., [53]).

It is important to note that Zadeh in his early papers considered shapes of fuzzy sets modeling the meaning of "small/medium/big" (or "young/medium age/old", etc.) in the form depicted in Fig. 1(a). From the very beginning of fuzzy control applications it turned out that these fuzzy sets are quite problematic when using the computational sup-min rule and that the triangular fuzzy sets of from Fig. 1(b) are more suitable and easy to be manipulated. The triangles were denoted by verbal labels VS (very small), SM (small), ME (medium), BI (big) and VB (very big) that gave the impression that natural language is involved. The fuzzy sets, however, are further modified to obtain the best control so that the intended meaning of the linguistic expressions that the authors originally had in mind is lost. Thus, despite the authors' proclaim that fuzzy control is based on expert knowledge expressed in natural language, the reality is different.

Explanation of this fact is easy: control engineers use, in fact, a system which provides efficient and mathematically well justified *approximation of a control function* (see, e.g., [6,9,29,41]) but not a logical inference rule that makes it possible to derive conclusion on the basis of information given in natural language. Because we seek the best approximation of a function, modification of shapes of fuzzy sets is necessary. We should realize, however, that the obtained model is *not linguistic* in the sense that semantics of natural language is never captured.

The fuzzy sets from Fig. 1(b) (or sets of fuzzy sets forming a fuzzy partition) regularly appear in many papers which have the adjective "linguistic" in their title. They present a fuzzy model of some process which, in essence, applies the Mamdani's method (see, e.g., [12,22,47] and many other papers). This step by step led to the impression that these fuzzy sets are proper model of semantics of the underlying linguistic expressions. We will below give arguments against such model. We will show that the triangular fuzzy sets (or sets of fuzzy sets forming a fuzzy partition), in fact, characterize typical values selected from extensions of these expressions and are a convenient means for generalization of categories used in psychological, sociological and other questionnaires investigating, e.g., quality of service, hotel, business, etc.

We argue that if the fuzzy set theory is to be generally accepted as a reasonable tool, which can be applied in modeling of the semantics of natural language, it is necessary to learn from the results of linguistic research. There are many papers (cf., e.g., [2,19,30,43,46,48]) in linguistic literature, which either directly analyze, or are closely related to the class of linguistic expressions which can be called *evaluative* and which are, in fact, considered in the literature on applications of fuzzy sets. In this paper we will show that fuzzy set theory and fuzzy logic indeed makes it possible to develop an advanced mathematical model of the semantics of evaluative linguistic expressions so that the developed algorithms behave as if "understanding" them.

The theory of evaluative expressions is a part of a wider program of *fuzzy natural logic* (FNL). Its goal is to develop a mathematical model of human reasoning whose typical feature is the use of natural language. One of the results of this model is an inference system that effectively works with evaluative expressions. It is called *Perception-based Logic Deduction* (PbLD) and is described, e.g., in [14,33,39].