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Progress on Fuzzy Mathematical Programming: A personal perspective

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Abstract

Fuzzy Linear Programming is among the best and most studied topics in the Fuzzy Sets and Systems area. In this paper, the author describes the main developments, results and solution methods achieved in this topic throughout the last 35 years, from a personal point of view. Short descriptions on different Fuzzy Linear Programming problems are provided and future research lines are briefly pointed out.

Keywords: Fuzzy Linear Programming; Fuzzy numbers; Soft computing

1. Introduction

The need for an optimal solution or the best solution among those available, in a properly proposed problem is the rationale behind studying the theories and proposing methodologies appropriate to the scientific field in which the problem arises. More specifically, although still a very broad area, there is an important type of problems, known as optimization problems, which are generally associated to finding the maximum or minimum value that a specific function can attain within a previously defined set. Everything related to such problems can be classified within the doctrinal field of Mathematical Programming, which covers a huge range of situations, whether they be linear cases, non-linear cases, randomness, single decision maker, several decision makers etc.

Among all of the models included in the Mathematical Programming field, the best and most studied is the single objective linear case (dealt with under Linear Programming), which has also been shown to have the greatest practical benefits. The methods and models of Linear Programming (LP) have useful applications in the areas of Engineering, Economy, Mathematics, Operations Research or Computational Intelligence as well as in other disciplines related to optimization to a greater or lesser degree. They constitute a more than suitable theoretical basis on which to tackle highly complex situations in an elegant and efficient manner. As occurs in the, let us say, conventional area

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of Mathematical Programming, also in the Fuzzy Sets and Systems area, the best and most studied problem of Fuzzy Mathematical Programming is that of Fuzzy Linear Programming (FLP).

While FLP has its theoretical precedents in 1970 in the great and seminal work on Decision Theory by R. Bellman and L.A. Zadeh [1], FLP problems were formally born in 1974, the year in which two separate papers [2,3] proposed the same model to deal with LP problems, i.e. that the set of constraints be given by a fuzzy set. Despite the coincidence, the works approached the solution from different points of view, and thus employed different methods, which led to a solution constituted by a single point, which therefore can be considered as outside the fuzzy context of the calculation. It was subsequently shown [4] that these two methods are particular cases of a more general method which allowed a context-dependent fuzzy solution, and which encompassed the solutions put forward in [2,3].

This paper is not intended to be a historical survey of FLP, but a description of the main results achieved by the author in this topic, and consequently it describes the path followed by the author on the topic of FLP from the early 80s up to the current date, paying special attention at the end of the work to possible future research areas that would deserve to be explored.

2. Contributions on FLP problems

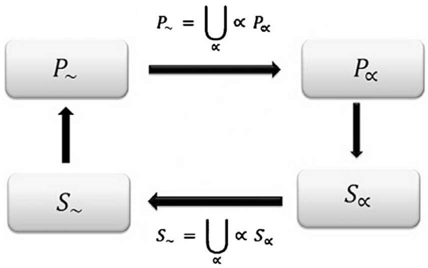
The first papers that the author of this work devoted to this topic were [5,6]. Among them, [5] should be highlighted because it was the starting point of a long list of works on this issue. Following the definitions from [2,3] and using conventional denotation, a Fuzzy Linear Programming (FLP) problem with fuzzy constraints can be formulated in the following way:

$$\begin{aligned} \text{Min:}\quad & z = cx \\ \text{s.t.:}\quad & Ax \leq^f b \\ & x \geq 0 \end{aligned} \qquad (P_\sim)$$

where the symbol "^{f}" indicates that the constraints are fuzzy ones, and hence the fuzziness is represented by membership functions defined by the decision maker. This means that the decision maker may permit some violations in the accomplishment of such restrictions.

The main idea for solving the FLP problem $P_\sim$ is to consider the α-cuts of the constraint set. Then, it is necessary to solve the (conventional) LP problem which appears at each α-cut, $P(\alpha)$, $\alpha \in [0, 1]$, by using conventional algorithms (Simplex Algorithm in the easiest case) to obtain an α-solution $S(\alpha)$. Hence, by using the Representation Theorem for Fuzzy Sets, finally all these $S(\alpha)$ solutions are integrated to give a final fuzzy solution $S_\sim$ to be associated to the former FLP problem $P_\sim$.

The solution method is described in the following scheme:



In essence, the methods suppose having to solve, for each $\alpha \in [0, 1]$, a conventional parametric LP problem as follows:

$$\begin{aligned} \text{Min:}\quad & z = cx \\ \text{s.t.:}\quad & Ax \leq b + d(1 - \alpha) \\ & x \geq 0, \alpha \in [0, 1] \end{aligned} \qquad (P_\alpha)$$

where "d" is the vector of values of the violations permitted, as the corresponding membership functions stand.