Available online at www.sciencedirect.com
ScienceDirect
Fuzzy Sets and Systems 255 (2014) 17–29
FUZZY sets and systems
www.elsevier.com/locate/fss

Fuzzy efficiency without convexity

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Received 15 February 2013; received in revised form 17 February 2014; accepted 11 April 2014

Available online 18 April 2014

Abstract

In this paper we develop a fuzzy version of the crisp Free Disposal Hull (FDH) method for measuring technical efficiency for samples of similar production units. The FDH-method is basically Data Envelopment Analysis (DEA) without the underlying assumption of convexity of the technology set. Our approach builds directly upon the definition of Farrell's indexes of technical efficiency used in crisp FDH. Therefore we do not require the use of fuzzy programming techniques but only utilize ranking probabilities of intervals as well as a related definition of dominance between pairs of intervals. We illustrate the approach using a data set of 200 Lithuanian family farms for the period of 2004–2009.

Keywords: Fuzzy efficiency; Fuzzy DEA; Fuzzy FDH; Non-convexity; Farm efficiency

1. Introduction

Efficiency measurement techniques are frequently applied on all levels of the economy. In particular, various versions of so-called Data Envelopment Analysis (DEA), have proved tremendously popular over the past couple of decades, see e.g., Cooper et al. [2], Bogetoft and Otto [1].

In practice, though, the results of DEA studies are often subject to considerable uncertainties. There are at least two main reasons for that:

First, the data that are used are typically connected with some level of uncertainty. This may not only be a result of stochastic measurement errors but also be caused by more systematic differences in data registration (for instance if units are compared across different countries or if units are compared across different time periods). In some countries data uncertainties are also caused by lack of tradition for data collection and thereby strong varying data quality with partly missing or estimated data etc.

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http://dx.doi.org/10.1016/j.fss.2014.04.009
0165-0114/

Second, the DEA methodology is very sensitive to such data uncertainties since the methods are non-parametric and based on extreme observations (undominated observations). Thus, flawed data distorts the estimated efficient frontier of the production possibility set, which plays the role as benchmark for all other observations.

These problems are well-recognized in the DEA-literature and dealt with in many different ways. One such way is to express data uncertainties through the use of fuzzy numbers instead of usual crisp data. Data sets consisting of fuzzy numbers can then be incorporated into the DEA framework as demonstrated in the, by now, substantial strand of literature on fuzzy DEA, see e.g., the recent survey in Hatami-Marbini et al. [8].

In this paper, we will suggest yet another approach particularly related to a variant of the DEA model called the FDH-method, see e.g. Deprins et al. [5], Tulkens [20]. The FDH-method is basically DEA without the assumption of convexity (of the technology). As such it builds on a minimal set of assumptions concerning the underlying production technology. While theoretically convenient, the convexity assumption is often questionable in practice.

We suggest a specific method for fuzzy production data which mimics the FDH-method for crisp data. For each α-level fuzzy data (in the form of fuzzy numbers) take the form of intervals. Our main idea is to rank such intervals using probabilities for having either the lowest or the highest value among a given set of intervals under the assumption that values are uniformly distributed over the range of each interval. Using these probabilities we define a dominance relation between production units as well as *max* and *min* operators over intervals. This enables us to mimic exactly the way (Farrell's radial technical) efficiencies are determined in the crisp FDH-model. The final fuzzy efficiency score combines the interval scores of each α-level set.

Our approach has the advantage to alternative approaches that all involved computations are quite simple. In this way we avoid, for instance, the use of fuzzy programming techniques. Moreover, our approach is quite flexible in the sense that it allows the analyst (and/or decision maker) to engage in an iterative decision process through changes in the various parameters of the method.

We illustrate the suggested fuzzy FDH-method using a data set on Lithuanian family farms where significant variation in data can be found for the same farm over several time periods. One reason for this can be stochastic events like changing weather conditions but other types of data uncertainties may be present as well. We therefore model data uncertainty by triangular fuzzy numbers where the value of a given variable for the year in question can be modeled as the kernel and the support is made up by respectively the minimal and maximal observation for that variable over the total time span. The result is a much more nuanced picture (with fuzzy efficiency scores) than what can be obtained by use of crisp FDH. The specific data set is selected from the Farm Accountancy Data Network (FADN) and covers a panel of 200 farms for the years 2004–2009.

1.1. Relation to the literature

Below we will point out some of the main approaches to fuzzy DEA and position our own contribution in the field.

There are at least four main ways to handle fuzzy data in connection with DEA. One is to use DEA efficiency scores together with absolute expert judgments to create fuzzy scores of efficiency, as suggested in Hougaard [9]. It is much more common, though, to use fuzzy data (fuzzy numbers or intervals) in connection with the mathematical programs of DEA. Most authors suggest to transform fuzzy DEA into crisp DEA for given level sets, see e.g. Triantis and Girod [17], Kao and Liu [13], Entani et al. [6], Wang et al. [21], Guo and Tanaka [7], León et al. [15] and Kao [12]. See also Hougaard [10] for a simple approximation of this type of approach. Another possibilistic approach treats the constraints of the DEA programs as fuzzy events, see e.g. Lertworasirikul et al. [16] while a fourth approach considers only pairwise dominance of fuzzy data, see e.g. Triantis and Vanden Eeckaut [19] and Triantis et al. [18].

The present method relates to the DEA model without the assumption of convexity (dubbed the FDH-method in [5]). Using the FDH-method to measure efficiency with interval production data can be done within the standard mathematical programming framework as shown in Jahanshahloo et al. [11]. This line of approach follows the spirit of so-called IDEA (DEA with imprecise data) introduced in Cooper et al. [3].

We do not rely on programming techniques. The present approach is therefore more related to the approach in [19] and [18] where the notion of dominance is defined for pairwise comparisons of fuzzy input and output data. Yet, in the present paper we define the dominance notion related to each α-level set using interval comparisons based on ranking probabilities. Simple computations together with these ranking probabilities enable us to mimic the calculation of traditional radial efficiency scores in the crisp FDH-model (see [20]) for fuzzy data resulting in fuzzy efficiency scores.