Available online at www.sciencedirect.com

ScienceDirect

Fuzzy Sets and Systems 255 (2014) 128–145

FUZZY sets and systems

www.elsevier.com/locate/fss

A Banzhaf value for games with fuzzy communication structure: Computing the power of the political groups in the European Parliament

I. Gallego, J.R. Fernández, A. Jiménez-Losada *, M. Ordóñez

Escuela Técnica Superior de Ingenieros, Matemática Aplicada II, Universidad de Sevilla, Camino de los Descubrimientos, 41092 Sevilla, Spain

Received 11 December 2012; received in revised form 14 November 2013; accepted 21 December 2013

Available online 30 December 2013

Abstract

In 2013, Jiménez-Losada et al. introduced several extensions of the Myerson value for games with fuzzy communication structure. In a fuzzy communication structure the membership of the players and the relations among them are leveled. Now we study a Banzhaf value for these situations. The Myerson model is followed to define the fuzzy graph Banzhaf value taking as base point the Choquet integral. We propose an axiomatization for this value introducing leveled amalgamation of players. An algorithm to calculate this value is provided and its complexity is studied. Finally we show an applied example computing by this fuzzy value the power of the groups in the European Parliament.
© 2014 Elsevier B.V. All rights reserved.

Keywords: Game theory; Fuzzy graphs; Banzhaf value; Power indices; European Parliament

Introduction

A cooperative game with transferable utility over a finite set of players is defined as a function establishing the worth of each coalition (subset of players). The outcome of a game is a payoff vector, namely it is a vector in which each component represents the payment for each player because of their cooperation possibilities. Although payoff vectors are usually efficient with regard to the worth of the great coalition, Owen [16] introduced the (probabilistic) Banzhaf value, based in the Banzhaf power index for simple games, as an interesting non-efficient outcome for each game. The usual payoff vectors suppose that all the communications are feasible. Myerson [12] considered that the communication among the players can be different at any time though the game was thought in a total cooperation situation. He described the communication situation by a graph where the vertices are the players and the links are the feasible bilateral communications among them. This graph is named the communication structure of the game. Hence we will use both, graph or communication structure, alike. So, Myerson proposed as a more realistic outcome for the game a payoff vector for each communication structure. A communication value assigns a payoff vector to each game with a specific communication structure. Owen [17] defined the graph Banzhaf value as a communication value

* Corresponding author.
E-mail address: ajlosada@us.es (A. Jiménez-Losada).

0165-0114/$ – see front matter © 2014 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.fss.2013.12.012

which coincides on the complete graph (the total cooperation situation) with the Banzhaf value for each game. Given a communication structure, the graph Banzhaf value calculates the Banzhaf value of the game modified by the graph. So, the worth of a coalition taking into account the graph is the sum of the worths of the maximal connected coalitions in the subgraph restricted by the corresponding set of vertices. Alonso-Meijide and Fiestras-Janeiro [1] characterized this communication value by three axioms: isolation (the payment of an isolated player is the worth of his individual coalition), fairness (the loss of one bilateral communication implies the same loss of payment for the players involved in this link) and pairwise merging (the payoff vector is neutral against pairwise mergers).

Aubin [2] and Butnariu [3] supposed uncertainty about the membership of the players in the coalitions studying games with fuzzy coalitions. To calculate the worth of a fuzzy coalition in a game it is necessary to consider a specific partition by levels of this fuzzy set. One of these partitions was defined by Tsurumi et al. [18] using the Choquet integral. Following this way, the uncertainty about the existence of the communications among the players can be extended to the uncertainty about these communications. Recently, Jiménez-Losada et al. [8] introduced fuzzy graphs to analyze communication among players. Fuzzy graphs allow leveling the links between being feasible or not, and they also allow considering membership levels for the players. The idea of partition by levels was extended to fuzzy communication structures in Jiménez-Losada et al. [9], proposing different extensions of the Myerson value for fuzzy situations. The main goal of this paper is to study the Banzhaf value for games with fuzzy communication structures extending the concept of graph Banzhaf value.

The legislative power in the European Union (EU) focuses on two chambers: the Council and the Parliament. The European Parliament is the elected body that represents the EU's citizens. It exercises political supervision over the EU's activities and takes part in the legislative process. Since 1979, members of the European Parliament have been directly elected, by universal suffrage, every five years. The Parliament takes part in the legislative work of the EU at three levels. The cooperation procedure (introduced in 1987 by the Single European Act): the European Parliament can give its opinion on draft directives and regulations proposed by the European Commission, which is asked to amend its proposals taking into account the Parliament's position. The assent procedure (also from 1987): the European Parliament must give its assent to international agreements negotiated by the Commission and to any proposed enlargement of the EU. The co-decision procedure (introduced in 1992 by the Treaty of Maastricht): the European Parliament is put on an equal footing with the Council when legislating on a whole series of important issues. In 2009, the Treaty of Lisbon gave to the European Parliament new legislative powers (over forty new items were included by the co-decision procedure). While the Council of the European Union represents the national governments of members states, the European Parliament pretends to be the ideologic representation of the European citizens. But actually the channel of voting is the set of national political parties in each member state. Hence, the relations among these groups are partial because of the national interests. Historically, the first paper on model analysis of the EU institutions, Holler and Kellermann [6], was focused on national distribution of voting power in the European Parliament (even before the first election in 1979). Hosli [7] and Noury [13] analyze voting power with national and ideological dimension of voting in the European Parliament. Later, other studies about the power in the European Parliament are Hix et al. [5] and Turnovec et al. [19]. Particularly, the Banzhaf index was used in Nurmi [15] to analyze the power of the political parties in the European Parliament.

In this paper we quantify the power of the political groups in the European Parliament taking into account their partial communications. The voting system in the Parliament is described by a weighted voting game, a specific simple cooperative game. For each voting in the chamber we can take, following Myerson [12], a graph representing the communication situation among the groups. We use our Banzhaf value for games with fuzzy communication structure to determine the power index of the groups in the European Parliament. The number of players in this game is too large to use the original formulas of the values. We show new algorithms to determine the indices and we study the time complexity of them.

Section 1 presents in short the background which allows the reader to follow the paper: cooperative games, games with communication structure, games with fuzzy communication structure. We define the fuzzy graph Banzhaf value following the Choquet by graph (cg) model in Section 2. We obtain in this section an axiomatization of the value. Section 3 is dedicated to the computation of cg-fuzzy graph Banzhaf value and the time complexity of the algorithm. Finally, in Section 4, we apply the values to study the power of the political groups in the European Parliament.

Download English Version:

https://daneshyari.com/en/article/389701

Download Persian Version:

https://daneshyari.com/article/389701

Daneshyari.com