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Bayesian abstract fuzzy economies, random quasi-variational inequalities with random fuzzy mappings and random fixed point theorems

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Abstract

The main purpose of this paper is to introduce an abstract fuzzy economy model with differential asymmetric information and a measure space of agents. It generalizes a previous model proposed by the author in 2009. The applications concern the fuzzy equilibrium existence and the existence of the solutions for two types of random quasi-variational inequalities. Furthermore, the paper presents random fixed point theorems with random fuzzy mappings, extensions of the ones with random data. Our results either generalize or improve corresponding ones present in literature. They consider the Bochner integrability setting, a measure space of indices and use random fuzzy mappings, which describe the uncertainties.

Keywords: Bayesian abstract fuzzy economy; Bayesian fuzzy equilibrium; Incomplete information; Random fixed point; Random quasi-variational inequalities; Random fuzzy mapping

1. Introduction

The concept of a fuzzy game (or a fuzzy abstract economy) has been introduced in [17] and the existence of the equilibrium for 1-person fuzzy game has been proven. The theory of fuzzy sets, initiated by Zadeh [41], has became a good framework for obtaining results concerning fuzzy equilibrium existence for fuzzy abstract economies. The existence of equilibrium points of fuzzy games has been also studied in [2,13–19,25–27,35].

Firstly, we introduce a fuzzy extension of Patriche's model of the Bayesian abstract economy [24] and we prove the existence of Bayesian fuzzy equilibrium. All economic activity in a society is made under conditions of uncertainty (incomplete information) and the model we propose captures this meaning. Actually, we define an abstract fuzzy economy with asymmetric information and a measure set of agents, each of which is characterized by a private information set, a fuzzy action (strategy) mapping, a random fuzzy constraint mapping and a random fuzzy preference mapping. The fuzzy equilibrium concept is an extension of the deterministic equilibrium. Our model generalizes the former deterministic ones introduced by Debreu [5], Shafer and Sonnenschein [31] or Yannelis and Prabhakar [37]

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or Patriche [24]. For an overview of results concerning the equilibrium of abstract economies, the reader should refer to [28].

As application, we study random quasi-variational inequalities with random fuzzy mappings. The variational inequalities were introduced in 1960s by Fichera and Stampacchia, who studied equilibrium problems arising from mechanics. Since then, this domain has been extensively studied and has been found very useful in many diverse fields of pure and applied sciences, such as mechanics, physics, optimization and control theory, operations research and several branches of engineering sciences. Huang introduced in [13] the concept of random fuzzy mappings and he studied in [15] the random generalized nonlinear variational inclusions for random fuzzy mappings. Recently, the random variational inequality and random quasi-variational inequality problems have been studied in [10–12,22,23,33,40]. Our results established in [29] and [30] concern the existence of the equilibrium for Bayesian fuzzy abstract economies with any set of agents and the applications to quasi-variational inequalities with random fuzzy mappings.

In this paper, we introduce an abstract fuzzy economy model with a measure space of agents which generalizes Patriche model [24], we prove a theorem of fuzzy equilibrium existence and prove the existence of the solution for two types of random quasi-variational inequalities with random fuzzy mappings. As a consequence, we obtain random fixed point theorems.

The paper is organized as follows. In the next section, some notational and terminological conventions are given. We also present, for the reader's convenience, some results on Bochner integration. In Section 3, the model of differential information abstract fuzzy economy and the main result are presented. Section 4 contains theorems concerning the existence of solutions for random quasi-variational inequalities with random fuzzy mappings.

2. Notation and definition

Throughout this paper, we shall use the following notation: $\mathrm{co}D$ denotes the convex hull of the set D. $\overline{\mathrm{co}}D$ denotes the closed convex hull of the set D. 2^D denotes the set of all non-empty subsets of the set D. If $D \subset Y$, where Y is a topological space, $\mathrm{cl}\, D$ denotes the closure of D.

For the reader's convenience, we review a few basic definitions and results from continuity and measurability of correspondences, Bochner integrable functions and the integral of a correspondence.

Let Z and Y be sets. The *graph* of the correspondence $P : Z \to 2^Y$ is the set $G_P = \{(z, y) \in Z \times Y\colon\ y \in P(z)\}$.

Let Z, Y be topological spaces and $P : Z \to 2^Y$ be a correspondence. P is said to be *upper semicontinuous* if for each $z \in Z$ and each open set V in Y with $P(z) \subset V$, there exists an open neighborhood U of z in Z such that $P(y) \subset V$ for each $y \in U$. P is said to be *lower semicontinuous* if for each $z \in Z$ and each open set V in Y with $P(z) \cap V \neq \emptyset$, there exists an open neighborhood U of z in Z such that $P(y) \cap V \neq \emptyset$ for each $y \in U$.

Lemma 1. *(See [39].) Let Z and Y be two topological spaces and let D be an open subset of Z. Suppose $P_1 : Z \to 2^Y$, $P_2 : Z \to 2^Y$ are upper semicontinuous such that $P_2(z) \subset P_1(z)$ for all $z \in D$. Then, the correspondence $P : Z \to 2^Y$ defined by*

$$P(z) = \begin{cases} P_1(z), & \text{if } z \notin D, \\ P_2(z), & \text{if } z \in D \end{cases}$$

is also upper semicontinuous.

Let E be a topological vector space and E' the dual space of E which consists of all continuous linear functionals on E. The real part of pairing between E' and E is denoted by $\mathrm{Re}\langle w, x\rangle$ for each $w \in E'$ and $x \in E$. The operator $P : E \to 2^{E'}$ is called *monotone* if $\mathrm{Re}\langle u - v, y - x\rangle \geqslant 0$ for all $u \in P(y)$ and $v \in P(x)$ and $x, y \in E$.

Let now (Ω, F, μ) be a complete, finite measure space, and Y be a topological space. The correspondence $P : \Omega \to 2^Y$ is said to have a *measurable graph* if $G_P \in F \otimes \beta(Y)$, where $\beta(Y)$ denotes the Borel σ-algebra on Y and $\otimes$ denotes the product σ-algebra. The correspondence $T : \Omega \to 2^Y$ is said to be *lower measurable* if for every open subset V of Y, the set $\{\omega \in \Omega\colon\ T(\omega) \cap V \neq \emptyset\}$ is an element of F. This notion of measurability is also called in the literature *weak measurability* or just *measurability*, in comparison with strong measurability: the correspondence $T : \Omega \to 2^Y$ is said to be *strong measurable* if for every closed subset V of Y, the set $\{\omega \in \Omega\colon\ T(\omega) \cap V \neq \emptyset\}$ is an element of F. In the framework we shall deal with (complete finite measure spaces), the two notions coincide