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New results on equilibria of abstract fuzzy economies

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Abstract

In this paper, we consider the model of an abstract fuzzy economy and we obtain new equilibrium theorems for the situations when the involved correspondences satisfy properties which are weaker than upper-semicontinuity. In our case, the constraint or preference correspondences are almost weakly upper semicontinuous or have the e-upper semicontinuous selection property. The obtained results either generalize or improve corresponding ones present in the literature.

Keywords: Almost weakly upper semicontinuous correspondences; Correspondences with e-USS property; Abstract fuzzy economy; Fuzzy equilibrium

1. Introduction

The concept of a fuzzy game (introduced in [10]) captures the idea that the individual feature of the decisions of the agents involved in different economic activities is characterized by uncertainties. The theory of fuzzy sets, initiated by Zadeh [23], and which had become a language of applied mathematics used in order to describe the phenomena which cannot be characterized precisely, proved to be a perfect framework for the study of the games having this type of characteristics.

The most important notion related to generalized fuzzy games (also called abstract fuzzy economies) is the fuzzy equilibrium and the study of its existence began with the paper of Kim and Lee [10], where, the existence of the equilibrium for 1-person fuzzy game was proven. After that, many published works have focused on finding conditions which can guarantee the equilibrium existence for new types of games in the fuzzy setting [8,9,11–16,19].

There are several generalizations of the classical model of abstract economy proposed in his pioneering works by Debreu [3] or later by Shafer and Sonnenschein [18], Yannelis and Prahbakar [21]. Yuan's model [22] was fuzzified and considered by several authors. Among them, we can quote Chang and Tan [2], Huang [8], Kim and Lee [11, 12] and Patriche [14,15]. There is a newer literature concerning other fuzzy models, as well. Huang studied in [9] the equilibrium existence for a generalized abstract fuzzy economy, Patriche [13] introduced the concept of the free abstract fuzzy economy. In this paper, we continue the study of the fuzzy equilibrium existence for the fuzzy extension of Yuan's model [22] in case when the conditions over correspondences, which are more general than the ones in the above-quoted articles, are assumed. By using a Wu-like [20] fixed point theorem for correspondences which fail to be

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upper semicontinuous, we prove our first result. On the other hand, we use a technique of approximation to prove an equilibrium existence theorem for set valued maps having e-upper semicontinuous selection property.

The paper is organized in the following way: Section 2 contains preliminaries and notation. The weakly upper semicontinuous correspondences with respect to a set and the fixed point theorem are presented in Section 3. The equilibrium theorems are stated in Section 4.

2. Preliminaries and notation

Throughout this paper, we shall use the following notation and definitions:

Let A be a subset of a topological space X. 2^A denotes the family of all subsets of A. cl$A$ denotes the closure of A in X. If A is a subset of a vector space, co$A$ denotes the convex hull of A. If $F, G : X \to 2^Y$ are correspondences, then co$G$, cl$G$, $G \cap F : X \to 2^Y$ are correspondences defined by $(\mathrm{co}G)(x) = \mathrm{co}G(x)$, $(\mathrm{cl}G)(x) = \mathrm{cl}G(x)$ and $(G \cap F)(x) = G(x) \cap F(x)$ for each $x \in X$, respectively. The graph of $T : X \to 2^Y$ is the set $\mathrm{Gr}(T) = \{(x, y) \in X \times Y \mid y \in T(x)\}$.

The correspondence $\overline{T}$ is defined by $\overline{T}(x) = \{y \in Y\colon (x, y) \in \mathrm{cl}_{X\times Y}\mathrm{Gr}(T)\}$ (the set $\mathrm{cl}_{X\times Y}\mathrm{Gr}(T)$ is called the adherence of the graph of T). It is easy to see that $\mathrm{cl}T(x) \subset \overline{T}(x)$ for each $x \in X$.

Notation. Let E and F be two Hausdorff topological vector spaces and $X \subset E$, $Y \subset F$ be two non-empty convex subsets. We denote by $\mathcal{F}(Y)$ the collection of fuzzy sets on Y. A mapping from X into $\mathcal{F}(Y)$ is called a fuzzy mapping. If $F : X \to \mathcal{F}(Y)$ is a fuzzy mapping, then for each $x \in X$, $F(x)$ (denoted by F_x in the sequel) is a fuzzy set in $\mathcal{F}(Y)$ and $F_x(y)$ is the degree of membership of point y in F_x.

A fuzzy mapping $F : X \to \mathcal{F}(Y)$ is called convex, if for each $x \in X$, the fuzzy set F_x on Y is a fuzzy convex set, i.e., for any $y_1, y_2 \in Y$, $t \in [0, 1]$, $F_x(ty_1 + (1 - t)y_2) \geqslant \min\{F_x(y_1), F_x(y_2)\}$.

In the sequel, we denote by

$$(A)_q = \{y \in Y\colon A(y) \geqslant q\}, \quad q \in [0, 1]$$

the q-cut set of $A \in \mathcal{F}(Y)$.

Definition 1. Let X, Y be topological spaces and $T : X \to 2^Y$ be a correspondence. T is said to be *upper semicontinuous* if for each $x \in X$ and each open set V in Y with $T(x) \subset V$, there exists an open neighborhood U of x in X such that $T(y) \subset V$ for each $y \in U$. T is said to be *almost upper semicontinuous* if for each $x \in X$ and each open set V in Y with $T(x) \subset V$, there exists an open neighborhood U of x in X such that $T(y) \subset \mathrm{cl}V$ for each $y \in U$.

Lemma 1. *(See Lemma 3.2, p. 94 in [24].) Let X be a topological space, Y be a topological linear space, and let $S : X \to 2^Y$ be an upper semicontinuous correspondence with compact values. Assume that the sets $C \subset Y$ and $K \subset Y$ are closed and respectively compact. Then $T : X \to 2^Y$ defined by $T(x) = (S(x) + C) \cap K$ for all $x \in X$ is upper semicontinuous.*

Lemma 2 is a version of Lemma 1.1 in [22] (for $D = Y$, we obtain Lemma 1.1 in [22]). For the reader's convenience, we include its proof below.

Lemma 2. *Let X be a topological space, Y be a nonempty subset of a locally convex topological vector space E and $T : X \to 2^Y$ be a correspondence. Let β be a basis of neighborhoods of 0 in E consisting of open absolutely convex symmetric sets. Let D be a compact subset of Y. If for each $V \in \beta$, the correspondence $T^V : X \to 2^Y$ is defined by $T^V(x) = (T(x) + V) \cap D$ for each $x \in X$, then $\bigcap_{V\in\beta} \overline{T^V}(x) \subseteq \overline{T}(x)$ for every $x \in X$.*

Proof. Let x and y be such that $y \in \bigcap_{V\in\beta} \overline{T^V}(x)$ and suppose, by way of contradiction, that $y \notin \overline{T}(x)$. This means that $(x, y) \notin \mathrm{clGr}(T)$, so that there exists an open neighborhood U of x and $V \in \beta$ such that:

$$\big(U \times (y + V)\big) \cap \mathrm{Gr}(T) = \emptyset. \tag{1}$$