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Importance analysis on the failure probability of the fuzzy and random system and its state dependent parameter solution

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Abstract

For structure systems with fuzzy input uncertainty as well as random one, the effects of the two kinds of uncertainties on the failure probability of the structure are studied, and an importance analysis model is established to quantitatively evaluate these effects. Based on the fact that the fuzziness of the output is determined by that of the input, the importance measure is defined to evaluate the effect of the fuzzy input variable. For the random input variable, the established model analyzes its importance from two aspects: (1) its effect on the most plausible value of the failure probability, (2) its effect on the imprecision of the failure probability. In the process of computing the importance measures, the conditional failure probability and unconditional one need to be evaluated, which is time demanding for practical engineering problems. For efficiently performing importance analysis in the presence of the fuzzy input variables and the random ones, the importance sampling (IS) method combined with the state dependent parameter (SDP) method is presented. Several examples show that the established importance analysis model can reflect the effects of the two kinds of input variables on the safety of the structure comprehensively and reasonably, and the presented solution can improve computational efficiency considerably with acceptable precision.

Keywords: Fuzzy input variable; Random input variable; Failure probability; Importance analysis model; Importance sampling; State dependent parameter method

1. Introduction

Sensitivity analysis (SA) aims at studying and analyzing the susceptibility of the output response of the model to the input parameters and surrounding conditions [1]. As an important branch of the SA, importance analysis, also known as global SA, of the input variables is significant in engineering design and probability safety assessment. Importance analysis aims at determining which of the input variables influence output the most in their whole uncertainty ranges. With the ranking of the input variables resulting from importance analysis, one can pay more attention or priority on the variables with high importance, and neglect the variables with low importance during the process of design and

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optimization. This can provide useful guidance for engineering design and optimization of the systems. At present, importance analysis approaches of the random variables have been fully developed, such as non-parametric techniques [2–4], variance-based importance measure indices and their solutions [5,6,1], moment independent importance measures [7–9], etc.

Nevertheless, these methods deal with random variables at the given probability distributions, whereas in engineering, the exact probability distributions of certain variables involved usually cannot be obtained due to scarce or even absent statistical information, i.e., certain variables may be subject to epistemic uncertainty. Differently from the random uncertainty which describes the nature or physical variability associated with a quantity of interest, the epistemic uncertainty results from lack of knowledge of fundamental phenomena, which can be reduced with the increasing recognition or additional observational data. In view of this, many non-probabilistic methods presently have been developed to represent epistemic uncertainty, such as interval analysis [10,11], convex modeling [12,13], and possibility theory and fuzzy set theory [14–17]. Among them, possibility theory has been proved as an effective tool for describing epistemic uncertainty.

Theory of possibility is based on two measures of confidence, possibility and necessity. They do not use additivity axiom, requiring confidence be defined on individual events, but are based on the attribution of confidence to subsets only of the sure event [14]. It has been shown that possibility theory can be used to define upper and lower bounds for probability measures compatible with the available data [16,17]. Fuzzy number theory then represents a very useful tool to perform operations in the framework of possibility theory. The main advantages of fuzzy analysis with respect to other methods are: it preserves the intrinsic random nature of most of physical variables even if it does not require the definition of their probability density distributions (PDFs); values with high or low confidence can be distinguished, where the term confidence has necessarily a completely different meaning with respect to PDFs defined in the framework of probability [18]. Based on the fuzzy number theory which models the epistemic uncertainty as a fuzzy variable represented by membership function, some methods have been developed for evaluating the importance of fuzzy variables [19–22]. With different measures, these methods can estimate the influence of the uncertainty of the fuzzy input variables on that of the fuzzy output from different aspects, but they considered only the fuzzy variables, instead of the mixture of random and fuzzy variables.

All the methods of importance analysis discussed above consider either random uncertainty represented by probability distribution or epistemic uncertainty described by membership function, whereas the practical problems often involve both types of uncertainties. This is because that in engineering some input variables of the computational model can be represented by probability distributions due to observed variability and sufficient statistics, while others are better represented by membership functions due to imprecision. In view of this, many "hybrid" methods have been proposed for joint propagation of these two kinds of uncertainties [23–28], and successful applications have been observed in some engineering fields, including those in civil [29], electronics [30], and aerospace engineering [31,32]. Although with proper treatment of the results [23,27,28], the amount of random variability and imprecision contained in the output can be quantified separately in these propagation methods, they cannot provide information on how the uncertainty in the output of the model can be apportioned to different sources of uncertainty in the model input, i.e. they cannot quantify the relative importance of the epistemic and random input variables.

As aforementioned, determining the relative importance of the input variables is of particular significance for engineering design and optimization. To perform importance analysis for models with mixed uncertain variables, two kinds of uncertainty importance measures of the structural reliability and response with respect to both fuzzy and random input variables are proposed in [33]. Yet this method only provides a total importance measure for one input variable, which does not distinguish the effects on the most plausible value and the imprecision of the output by one input variable. Furthermore, the computational method for the importance measures in [33] still relies on the computationally expensive double-loop nested sampling procedure. An uncertainty importance measure system and its efficient point estimates solution are presented in [34] for evaluating the effects of the fuzzy and random input variables on the expectation and variance of the model output. By defining different importance measures, this method can analyze the importance of the fuzzy and random input variables comprehensively. Nevertheless, in the reliability analysis, most small failure probability is related to tail distribution of the model output, the effect of the input variable on the moments of the model output is not equal to that on the failure probability. Besides, estimating failure probability by the method of point estimates (i.e. moment method) is prone to wrong.

To evaluate the effects of the epistemic and random input variables on the structural failure concerned in the reliability engineering, a new importance analysis model is established in this paper, by extending the importance