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Fuzzy differential equation with completely correlated parameters

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Abstract

In this paper we study fuzzy differential equations with parameters and initial conditions interactive. The interactivity is given by means of the concept of completely correlated fuzzy numbers. We consider the problem in two different ways: the first by using a family of differential inclusions; in the second the extension principle for completely correlated fuzzy numbers is employed to obtain the solution of the model. We conclude that the solutions of the fuzzy differential equations obtained by these two approaches are the same. The solutions are illustrated with the radioactive decay model where the initial condition and the decay rate are completely correlated fuzzy numbers. We also present an extension principle for completely correlated fuzzy numbers and we show that Nguyen's theorem remains valid in this environment. In addition, we compare the solution via extension principle of the fuzzy differential equation when the parameters are non-interactive fuzzy numbers and when the parameters are completely correlated fuzzy numbers. Finally, we study the SI-epidemiological model in two forms: first considering that the susceptible and infected individuals are completely correlated and then assuming that the transfer rate and the initial conditions are completely correlated.

Keywords: Completely correlated fuzzy numbers; Extension principle; Fuzzy differential inclusion; Fuzzy differential equation; SI-model

1. Introduction

We begin by considering the initial value problem

$$\begin{cases} x'(t) = f\big(x(t), w\big) \\ x(t_0) = x_0, \end{cases} \tag{1}$$

where $x_0, w \in \mathbb{R}$ and f is a continuous function.

By making the change of variables $y = (x, w)$, the initial value problem (1) may be rewritten as

$$\begin{cases} y'(t) = F\big(y(t)\big) \\ y(t_0) = y_0, \end{cases} \tag{2}$$

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where $F(y(t)) = (f(x(t), w), 0)$ and $y_0 = (x_0, w)$.

Supposing that x_0 and w are uncertain and given by completely correlated fuzzy numbers X_0 and W, y_0 can be modeled by the joint possibility distribution C of X_0 and W (see Section 2). So, it is possible to relate the deterministic problem (2) with the fuzzy initial value problem

$$\begin{cases} y'(t) = F\big(y(t)\big) \\ y(t_0) \in C. \end{cases} \tag{3}$$

The problem (3) is merely a notation, which can be approached in two different ways. The first approach is suggested by Hüllermeier [1], in which the solution is given by the following family of differential inclusions:

$$\begin{cases} y'(t) = F\big(y(t)\big) \\ y(t_0) \in [C]^\alpha, \end{cases} \tag{4}$$

where $[C]^\alpha$ are the α-levels of the fuzzy subset C.

The second approach is obtained by means of the application of the extension principle for completely correlated fuzzy numbers to the deterministic solution of the problem (2) as made in [2,3]. In Mizukoshi et al. [2] the study was made for the case that the joint possibility distribution C is given by the minimum t-norm of two fuzzy numbers, that is, the fuzzy numbers are non-interactive.

Oberguggenberger and Pittschmann in [3] studied Eq. (1) for the case in which the coefficients and initial conditions are fuzzy. They defined the operators fuzzy equation, fuzzy restriction and fuzzy solution of Eq. (1) and apply the Zadeh's extension principle in these operators as model for obtaining the solution of the fuzzy equation associated with Eq. (1). Besides, they established the concepts of fuzzy solution and for parts component fuzzy solution for the fuzzy differential equation in subject.

Bede and Gal in [4] presented a new approach to solve fuzzy differential equations. They introduced and studied a new generalized concept of differentiability which extends usual known concept of differentiability of fuzzy-number-valued functions [5–7]. Following this idea, Chalco-Cano and Román-Flores [8] introduced the concept of fuzzy lateral H-derivative for fuzzy mapping and also demonstrated that the fuzzy solution is equivalent to the solution via differential inclusion [9,1].

J. Baetens and B. Baets [10] applied the concept of completely correlated fuzzy number for a discrete model to study the evolution of a disease in space and time. Those approaches have been recently extended to the case where two kinds of uncertainties, fuzziness and randomness, are present on the phenomenon. However, we do not threat this case here. This theory is called fuzzy stochastic differential equations, which is based on stochastic differential equations. The reader can refer to [11–14].

In Section 2, we define the extension principle for completely correlated fuzzy numbers and show that Nguyen's theorem [15] remains valid. In Section 3, we show that a solution of (3) can be obtained through extension principle, similarly to [2,3]. We conclude that this fuzzy solution coincides with the solution obtained by using Hüllermeier's approach, via differential inclusions. In order to exemplify we study the radioactive decay model with fuzzy intrinsic rate of decay and fuzzy initial conditions. In Section 4, we apply the studied methods to the SI-model with susceptible individuals completely correlated with infected individuals and also considering the transfer rate completely correlated with the initial conditions.

2. Basic concepts

We denote by $\mathcal{K}^n$ the family of all the non-empty compact subsets of $\mathbb{R}^n$. For $A, B \in \mathcal{K}^n$ and $\lambda \in \mathbb{R}$ the operations of addition and scalar multiplication are defined by

$$A + B = \{a + b : a \in A, b \in B\} \quad \text{and} \quad \lambda A = \{\lambda a : a \in A\}.$$

A fuzzy subset A of $\mathbb{R}^n$ is given by a function

$$\mu_A : \mathbb{R}^n \longrightarrow [0, 1].$$

The α-levels or α-cuts of the fuzzy subset A are defined by

$$[A]^\alpha = \big\{x \in \mathbb{R}^n : \mu_A(x) \geq \alpha\big\} \quad \text{for } 0 < \alpha \leq 1.$$