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Short communication

A note on "A class of linear differential dynamical systems with fuzzy initial condition"

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Abstract

In this note, we show that the proposed approach in J. Xu et al. (2007) [1] fails to obtain the stable solutions to the stable linear dynamical systems with fuzzy initial conditions. As it will be explained, this shortcoming is caused by neglecting to move the stability property of the fuzzy system to the quasi-level-wise system. Moreover, to handle this, another approach is proposed similar to the previous one for the stable and unstable systems.

Keywords: Fuzzy differential equations; Stability property; Quasi-level-wise system

1. Introduction and basic concepts

Throughout this note, the set of all real numbers is denoted by $\mathbb{R}$, and the set of all type-1 fuzzy numbers on $\mathbb{R}$ by E_1. The left and right end-points of the α-*level* sets of $\tilde{A}$, $[\tilde{A}]^\alpha$, are denoted by $\underline{A}^\alpha$ and $\bar{A}^\alpha$ respectively. J. Xu et al. in [1] proposed an approach to solve a class of fuzzy differential equations [FDEs] system. The approach is based on a complex number representation of the α-*level* sets of the fuzzy system. Let this kind of representation be called quasi-level-wise system. The following class of FDEs system has been considered in [1]

$$\dot{\tilde{X}}(t) = A\tilde{X}(t) + \tilde{f}(t), \qquad \tilde{X}(0) = \tilde{X}_0 \tag{1.1}$$

where $\dot{\tilde{X}}(t) = \frac{d\tilde{X}(t)}{dt}$, $\tilde{X} : [0, t_f] \to E_1^n$ is a fuzzy function on the real interval $[0, t_f]$, $A = [a_{ij}]_{n\times n}$ is a matrix, $a_{ij} \in \mathbb{R}$, $\tilde{f} : [0, t_f] \to E_1^n$ is a fuzzy function, and $\tilde{X}_0$ is the initial value of the system. Using the approach as in [1], the authors have obtained an unstable solution to a stable system, see Example 4.6 in [1]. In this note, we are going to show how this shortcoming stems from neglecting to move the stability property of the fuzzy system to the quasi-level-wise

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system. Moreover, to handle this, another approach is proposed similar to the previous one for the stable and unstable systems.

Consider the Linear Time Invariant [LTI] system $\dot{X}(t) = AX(t)$, where $X : [0, t_f] \to \mathbb{R}^n$, and $A = [a_{ij}]_{n\times n}$, $a_{ij} \in \mathbb{R}$. The eigenvalues of A, λ_i, $i = 1, ..., n$, are the roots of the characteristic polynomial $\det(\lambda_i I - A)$, $i = 1, ..., n$, where $\det(\cdot)$ means determinant and I is the $n \times n$ identity matrix. Now we are going to recall the following well-known definition and theorem.

Definition 1.1. The system $\dot{X}(t) = AX(t) + U(t)$ is said to be stable if for any bounded input $U(t) = [u_i(t)]_{n\times 1}$ the output $y(t) = [c_1...c_n]X(t)$ is bounded.

Theorem 1.1. *The LTI system $\dot{X}(t) = AX(t)$ is asymptotically stable if and only if the real parts of the eigenvalues of A are negative.*

Note 1.1. If the LTI system $\dot{X}(t) = AX(t)$ is asymptotically stable, then the system $\dot{X}(t) = AX(t) + U(t)$ is stable provided that $U(t)$ is bounded.

2. Proposed approach and example

In this section, an approach similar to that in [1] is proposed to obtain the solutions of FDEs system (1.1). It is assumed that $\tilde{f}(t)$ is bounded likewise, all of eigenvalues of A are also just either real negative or real positive. We recall, based on Theorem 5 in [2], the solution to the following FDE

$$\dot{\tilde{x}}(t) = -\tilde{x}(t), \qquad \tilde{x}(0) = \tilde{x}_0 \tag{2.1}$$

is differentiable in the second form, i.e., $[\dot{\tilde{x}}(t)]^\alpha = [\dot{\bar{x}}^\alpha(t), \dot{\underline{x}}^\alpha(t)]$. It should be noted that the eigenvalue of fuzzy system (2.1) is -1, and then obviously $\tilde{x}(t)$ is stable. Using the approach in [1], to obtain the solution to (2.1), the following system needs to be solved

$$\dot{\underline{y}}^\alpha(t) + i\dot{\bar{y}}^\alpha(t) = -\bar{y}^\alpha(t) - i\underline{y}^\alpha(t), \qquad \bar{y}^\alpha(0) = \bar{x}_0^\alpha, \qquad \underline{y}^\alpha(0) = \underline{x}_0^\alpha \tag{2.2}$$

The eigenvalues corresponding to (2.2) are -1 and $+1$ which means the system can be unstable, while the fuzzy system (2.1) is always stable. This contradiction is caused by the point that in moving from an FDEs system to the quasi-level-wise system, in addition to the formal structure of the FDEs system, the inherent properties of the variables also need to be moved.

In moving from an FDEs system to the quasi-level-wise system, using operators "g" and "e" is needed. Consider the FDE (2.1), by neglecting to move the stability property of $\tilde{x}(t)$, i.e. moving the instability property to the level-wise system; we get to the same system as (2.2)

$$e\big(\dot{\underline{y}}^\alpha(t) + i\dot{\bar{y}}^\alpha(t)\big) = -g\big(\underline{y}^\alpha(t) + i\bar{y}^\alpha(t)\big), \qquad \bar{y}^\alpha(0) = \bar{x}_0^\alpha, \qquad \underline{y}^\alpha(0) = \underline{x}_0^\alpha \tag{2.3}$$

Using the operator "g" we have $g(\dot{\underline{y}}^\alpha(t) + i\dot{\bar{y}}^\alpha(t)) = \dot{\bar{y}}^\alpha(t) + i\dot{\underline{y}}^\alpha(t) = \frac{d(g(\underline{y}^\alpha(t)+i\bar{y}^\alpha(t)))}{dt}$, and the stability property can be moved to the level-wise system as follows:

$$g\big(\dot{\underline{y}}^\alpha(t) + i\dot{\bar{y}}^\alpha(t)\big) = -g\big(\underline{y}^\alpha(t) + i\bar{y}^\alpha(t)\big), \qquad \bar{y}^\alpha(0) = \bar{x}_0^\alpha, \qquad \underline{y}^\alpha(0) = \underline{x}_0^\alpha \tag{2.4}$$

applying the operators leads to

$$\dot{\bar{y}}^\alpha(t) + i\dot{\underline{y}}^\alpha(t) = -\bar{y}^\alpha(t) - i\underline{y}^\alpha(t), \qquad \bar{y}^\alpha(0) = \bar{x}_0^\alpha, \qquad \underline{y}^\alpha(0) = \underline{x}_0^\alpha \tag{2.5}$$

The system (2.5) has two eigenvalues -1, -1 and is always stable. Then, the stability property is preserved by applying the operator "g".

Consequently, unlike the results mentioned in conclusions in [1], the properties of classical differential equations system are not lost if the coefficients matrix A is not assumed to be non-negative, but it is the applied approach to blame. Beyond doubt, neglecting the stability or instability property of the FDEs system variables, i.e. not moving them the quasi-level-wise system will lead to solutions that not only are misleading, but also may behave differently from the natural behavior of the main system.