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Fuzzy Sets and Systems 243 (2014) 1–24

FUZZY sets and systems

www.elsevier.com/locate/fss

A spectral approach for fuzzy uncertainty propagation in finite element analysis

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Received 13 November 2011; received in revised form 8 October 2013; accepted 9 October 2013

Available online 16 October 2013

Abstract

Uncertainty propagation in complex engineering systems with fuzzy variables constitutes a significant challenge. This paper proposes a Polynomial Chaos type spectral approach based on orthogonal function expansion. A fuzzy variable is represented as a set of interval variables via the membership function. The interval variables are further transformed into the standard interval $[-1, 1]$. Smooth nonlinear functions of standard interval variables are projected in the basis of Legendre polynomials by exploiting its orthogonal properties over the interval $[-1, 1]$. The coefficients associated with the basis functions are obtained by a Galerkin type of error minimisation. The method is first illustrated using scalar functions of multiple fuzzy variables. Later the method is proposed for elliptic type finite element problems where the technique is extended to vector valued functions with multiple fuzzy variables. The response of such systems can be expressed in the complete basis of multivariate Legendre polynomials. The coefficients, obtained by Galerkin type of error minimisation, can be calculated from the solution of an extended set of linear algebraic equations. An eigenfunction based model reduction technique is proposed to obtain the coefficient vectors in an efficient way. A numerical example of axial deformation of a rod with fuzzy axial stiffness is considered to illustrate the proposed methods. Linear and nonlinear membership functions are used and the results are compared with direct numerical simulation results.

Keywords: Fuzzy systems; Structural mechanics; Uncertainty propagation; Polynomial chaos; Spectral approach

1. Introduction

Finite element method [1] has been used widely for numerical simulation of complex engineering systems. The consideration of uncertainties in numerical models to obtain the variability of response is becoming more common for finite element models arising in practical problems. When substantial statistical information exists, the theory of probability and stochastic processes offer a rich mathematical framework to represent such uncertainties. In a probabilistic setting, uncertainty associated with the system parameters, such as the geometric properties and constitutive relations (i.e. Young's modulus, mass density, Poisson's ratio, damping coefficients), can be modeled as random variables or stochastic processes using the so-called parametric approach. These uncertainties can be quantified and propagated, for example, using the stochastic finite element method [2–4]. The reliable application of probabilistic approaches

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0165-0114/$ – see front matter
http://dx.doi.org/10.1016/j.fss.2013.10.005

requires information to construct the probability density functions of uncertain parameters. This information may not be easily available for many complex practical problems. In such situations, non-probabilistic approaches such as interval algebra [5], convex models [6] and fuzzy set [7] based methods can be used. In this paper the uncertain variables describing the system parameters are modeled using fuzzy variables.

Fuzzy finite element analysis (see for example the review papers [8–10]) aims to combine the power of finite element method and uncertainty modelling capability of fuzzy variables. One way to view a fuzzy variable is the generalisation of an interval variable. It should be noted that the intervals do not represent the values of the variable, but our knowledge about the range of possible values the variable can take. When an uncertain variable is modelled using the interval approach, the values of the variable lie within a lower and an upper bound. The fuzzy approach generalises this concept by introducing a membership function. In the context of computational mechanics, the aim of a fuzzy finite element analysis is to obtain the membership function of the output quantities (such as displacement, acceleration and stress) given the membership of data in the set of input variables. This problem, known as the uncertainty propagation problem, has taken the centre stage in recent research activities in the field. In principle an uncertainty propagation problem can be always solved using the so-called direct Monte Carlo simulation. Using this approach, a large number of members of the parameter set are individually simulated and bounds are obtained from the resulting outputs. For the most practical problems, direct Monte Carlo simulation is prohibitively computationally expensive. Therefore, the aim of the majority of current research is to reduce the computational cost. Under the possibilistic interpretation of fuzzy sets and using the min-operator as t-norm [11,12], fuzzy variables would become a generalisation of interval variables. Consequently methods applicable for interval analysis such as classical interval arithmetic [5], affine analysis [13] or vertex theorems [14] can be used. The Neumann expansion [15], the transformation method [16], and more recently, response surface based methods [17] have been proposed for fuzzy uncertainty propagation. In the context of structural dynamical systems, several authors have extended the classical modal analysis to fuzzy modal analysis [18–22]. Fuzzy approach has been applied to safety analysis [23,24], optimal design [25] and also boundary element analysis [26]. Recently a High Dimensional Model Representation (HDMR) approach has been proposed [27] for the propagation of fuzzy uncertain variables through a complex finite element model.

In spite of these significant developments in the methodology of fuzzy uncertainty propagation through complex systems, probabilistic uncertainty propagation methodologies perhaps still have the edge due to the availability of a wide-ranging techniques suited for various problems. Among the various available methods, the spectral methods [2] have received considerable attention [28]. These methods, generally known as polynomial chaos [29], stand on rigorous mathematical footing and provide a practical computational approach which is general enough to be used within the context of finite element method for complex systems [2]. Although the probabilistic approaches are powerful tools for reliability analysis in structural engineering design [30], these approaches usually require large volumes of data. However when limited information is available, the probabilistic approaches lose their performance significantly and might not be reliable to be used on their own. Fuzzy methods are found to have a better performance compared to probabilistic approaches in the face of lack of information [31]. Therefore, using a mixture of different uncertainty models in complex engineering systems may be useful, as neither the fuzzy description nor the probabilistic description may be available for *all* the uncertain parameters. Currently different mathematical techniques are being used for fuzzy and probabilistic variables. In this paper we propose a new technique by which spectral methods originally introduced for probabilistic uncertainty propagation, can be extended for fuzzy uncertainty propagation with suitable adaptations. The main motivation behind this work arises from computational efficiency, generality and rigour behind the fuzzy uncertainty propagation technique and also having a synergy between probabilistic and fuzzy approaches. Such synergy in the propagation technique between different uncertainty modelling approaches may lead to unified approaches whereby different types of uncertainty can be mathematically handled simultaneously in a consistent manner. Our approach relies on the decomposition of a fuzzy variable into parameterised interval variables and projecting the response function of these variables into a complete orthonormal basis of polynomial functions. The coefficients associated with the basis functions are obtained by a Galerkin type of error minimisation.

The outline of the paper is as follows. Section 2 gives a brief background of fuzzy variables. Legendre polynomials and its application for propagation of fuzzy variables through general nonlinear scalar functions are discussed in Section 3. Two simple numerical examples are given to illustrate the idea behind the proposed spectral approach. In Section 4 spectral finite element analysis with fuzzy variables for elliptic problems is proposed. A reduced computational approach is proposed in Section 4.3. A numerical example of axial deformation of a rod with fuzzy axial stiffness is given in Section 5 to illustrate the proposed methods. Different types of membership functions are used