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# A fuzzy syllogistic reasoning schema for generalized quantifiers

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**Abstract**

In this paper, a new approximate syllogistic reasoning schema is described that expands some of the approaches expounded in the literature into two ways: (i) a number of different types of quantifiers (logical, absolute, proportional, comparative and exception) taken from *Theory of Generalized Quantifiers* and similarity quantifiers, taken from statistics, are considered and (ii) any number of premises can be taken into account within the reasoning process. Furthermore, a systematic reasoning procedure to solve the syllogism is also proposed, interpreting it as an equivalent mathematical optimization problem, where the premises constitute the constraints of the searching space for the quantifier in the conclusion.


*Keywords:* Fuzzy syllogism; Fuzzy quantifiers; Approximate reasoning

## 1. Introduction

Human beings usually manage statements or propositions that involve quantities that are more or less well defined. Quantifiers, such as *all*, *few*, 25, *around* 25, …, are the linguistic particles frequently used to express them. Quantified statements (sentences involving quantifiers) are used both for describing particular aspects of reality (i.e., *Most students are young*) and also for making inferences; that is, obtaining new information from a given set of premises. This type of reasoning is known as *syllogism*. Although syllogisms were superseded by propositional logic [1] in the 19th century, it is still matter of research. In this paper, we propose an approach to syllogism involving fuzzy generalized quantifiers that allows the reasoning to be performed with no limits in the number of premises.

Table 1 shows an example of an usual syllogism, made up of two premises and one conclusion. In this case the premises are the quantified statements denoted as $PR1$ (first premise) and $PR2$ (second premise) and the conclusion is the quantified statement denoted as $C$ (inferred from the premises). Each one of the quantified statements in the example is made up of two main elements: a quantifier (*eight*, *ten*, *eight or less*) and terms (*students*, *Portuguese*, *young*, *Portuguese and young*), usually interpreted as sets, that describe properties of the elements in a referential universe. The most usual quantified statements are the binary ones; that is, statements involving a single quantifier and two terms. The subject in the sentence is the "restriction" of the quantifier and the predicate its "scope". For instance,

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Table 1
An example of syllogism.

| | |
|---|---|
| $PR1$ | Eight students are Portuguese |
| $PR2$ | Ten students are young |
| $C$ | Eight or less students are Portuguese and young |

*students* is the restriction and *Portuguese* the scope of quantifier *eight* in $PR1$ premise of Table 1. The purpose of inference within this context is to calculate consistent values for the quantifier in the conclusion starting from the premises. These values are strongly dependent on the actual distribution of the elements in the referential among the terms.[1] In the example, the consistent values for the quantifier in the conclusion range from 0, for the case in which none of the eight Portuguese students is young, to 8, for the case in which all of the Portuguese students are also young.

Syllogistic reasoning is an interesting field from different points of view that include theoretical and applied ones. From a theoretical perspective, as a logic, it is a kind of reasoning that should be analysed and understood in order to increase our knowledge about how human beings perform common-sense reasoning in daily life. From an applied perspective, it is an interesting tool for the fields of decision making or database systems [3]. Furthermore, our approach to syllogistic reasoning can be contextualized in the Computing with words [4] and Computational Theory of Perceptions [5] paradigms, because we try to manage a form of common-sense reasoning preserving the natural language surface. On the other hand, it is relevant to note that, from the point of view of reasoning, we usually focus on situations where quantified statements are assumed to be true and we are only concerned about what types of information can be inferred from them [6]. A different problem is the evaluation of quantified statements where the information about the fulfillment of the involved terms in a given universe is available. Models for this problem [7,8] do not consider the topic of syllogistic reasoning.

The first systematic approach to syllogistic reasoning was developed in [9]. Nevertheless, this framework only deals with arguments composed of two premises and one conclusion and only handles the four classical logic quantifiers (*all*, *none*, *some*, *not all*). Therefore, expressiveness of this model is limited to simple statements that are far from the common uses of language.

Most of the approaches found in the literature try to extend syllogistic reasoning in two parallel ways:

- by adding new quantifiers (like *most*, *few*, *many*, and so on) to the classical ones, considering crisp [10] and fuzzy [11–16] definitions. In these approaches only absolute/relative quantifiers involving two terms are handled, and many of the classical syllogistic reasoning patterns are not even considered [17]
- by considering arguments composed of $N$ statements and $N$ terms but limited to the four logic quantifiers [18], therefore missing much of the actual expressive necessities of daily language and reasoning.

These two ways have been approached in a mutually exclusive way, probably due to the fact that the first approaches come from the field of fuzzy modeling and the second ones from the field of linguistics and natural logic. To the best of our knowledge, no combination of both points of view has been made so far for providing a meaningful extension of the syllogistic patterns that at the same time involve relevant quantifiers (as those described in the linguistics field) and more than two terms in the statements. Therefore a general approach to syllogistic reasoning remains an incomplete task, since these approaches deal with syllogistics only from partial perspectives. Within this context, the main aim of this paper is to present a general formulation of syllogistic reasoning and its resolution that can be useful for all the fields aforementioned. Our formulation of syllogism is capable of managing different types of quantifiers within the *Theory of Generalized Quantifiers* (TGQ) (going much further than the usual absolute/relative quantifiers) and more than two premises in the reasoning scheme. Thus, our model can deal with more complex arguments and more wide fragments of natural language. Notwithstanding, the combination of absolute/proportional quantifiers in the same syllogism remains an open question and it is not dealt with in this paper.

The structure of the paper is as follows: in Section 2 we present the most relevant approaches to fuzzy syllogistic reasoning; in Section 3 the general form of our proposal of syllogistic schema and its resolution are detailed; in Section 4 some illustrative examples are shown. Finally, in Section 5, we summarize the conclusions of the paper.

[1] In [2], an extension of Aristotelian syllogistics considering the problem of the distribution is proposed.