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On the construction of interval-valued fuzzy morphological operators

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Abstract

Classical fuzzy mathematical morphology is one of the extensions of original binary morphology to greyscale morphology. Recently, this theory was further extended to interval-valued fuzzy mathematical morphology by allowing uncertainty in the grey values of the image and the structuring element. In this paper, we investigate the construction of increasing interval-valued fuzzy operators from their binary counterparts and work this out in more detail for the morphological operators, which results in a nice theoretical link between binary and interval-valued fuzzy mathematical morphology. The investigation is done both in the general continuous and the practical discrete case. It will be seen that the characterization of the supremum in the discrete case leads to stronger relationships than in the continuous case.
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1. Introduction

The image processing domain consists of numerous theories to extract information (edges, patterns, etc.) from images. Mathematical morphology is among these theories. In this theory [1–3], images are elements of a complete lattice (L, C) with L the space and C the comparison operator. Morphological operators have certain properties with respect to the comparison operator. For instance, dilations commute with supremum. Many morphological operators transform images with the help of subsets called structuring elements. Originally, only binary (black and white) images and structuring elements were considered. Next, binary morphology [4] was extended to greyscale images in three different ways. In the *threshold approach* [4], the structuring element still had to be binary, while in the *umbra approach* [5], also greyscale structuring elements were allowed. A third approach, *fuzzy mathematical morphology* [6], was introduced some time later and was inspired by the observation that both greyscale images and fuzzy sets are

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modelled as mappings from a universe $\mathcal{U}$ into the unit interval [0,1]. Fuzzy set theory is thus used as a tool here and not to model uncertainty. Besides the extensions from greyscale morphology to multivalued morphology (e.g. [7,8]), recently, greyscale fuzzy mathematical morphology was also further extended based on extensions of fuzzy sets to be able to deal with uncertain and bipolar information [9–13]. The interval-valued fuzzy extension introduced in [12,13], on which we concentrate in this paper, now has the important feature that it is no longer only used as a tool, but also as a model to represent uncertainty regarding the measured grey levels. In this model, a pixel in the image domain is no longer mapped onto one specific grey value ($\in [0, 1]$), but onto an interval of grey values to which the uncertain grey value is expected to belong. For a discussion on the interval-valued image model, we refer to [12,19].

As a side note, we mention here that interval-valued fuzzy set theory and intuitionistic fuzzy set theory [14] are equivalent [15] and as a consequence also interval-valued and intuitionistic fuzzy mathematical morphology. So the results in this paper can be translated to the intuitionistic/bipolar model [9,10] straightforwardly.

Finally, we remark that interval-valued representations also occur in a natural way in other image processing subdomains. Examples can be found in inverse halftoning [16], in the context of wavelets [17] and in edge detection applications [18]. In the latter example, the interval-valued representation is however rather a tool than a model.

In our previous work [19], we already studied the decomposition of the interval-valued fuzzy morphological operators into their cuts. More precisely, we investigated the relationships between the cuts of an interval-valued fuzzy morphological operator and the corresponding binary operator applied on the cuts of the image and structuring element. In this paper, we tackle the reverse problem, i.e. the construction of interval-valued fuzzy morphological operators from the binary operators. We start from a more general perspective, in which we investigate the construction of increasing interval-valued fuzzy operators from binary ones and additionally apply the construction principle on the binary dilation that is increasing w.r.t. both the image and structuring element.

This is first of all interesting from a theoretical point of view since it provides us a link between binary and interval-valued fuzzy mathematical morphology. It allows us to compute the interval-valued operators by combining the results of several binary operators and also to approximate them by a finite number of binary operators. We perform this investigation both in the general continuous framework and the practical discrete framework. In practice, image domains and allowed grey values have namely been sampled down due to technical limitations. As will be shown, some stronger relationships hold in this discrete case.

The paper is organized as follows. We repeat the basic principles of interval-valued fuzzy mathematical morphology in Section 2. Section 3 then studies the construction of the interval-valued fuzzy morphological operators based on weak and strict $[\alpha_1, \alpha_2]$-cuts in a continuous framework. Section 4 deals with the construction in a discrete framework, which leads to slightly different results. Section 5 concludes the paper.

2. Interval-valued fuzzy mathematical morphology

An interval-valued fuzzy set [20] is an extension of a classical fuzzy set [21]. Whereas a fuzzy set F in a universe $\mathcal{U}$ maps every element $u \in \mathcal{U}$ onto its membership degree $F(u) \in [0, 1]$ in the set F, an interval-valued fuzzy set G in the universe $\mathcal{U}$ maps every $u \in \mathcal{U}$ onto a closed subinterval $G(u) = [G_1(u), G_2(u)]$ of [0,1], in this way allowing uncertainty about the membership degree. An interval-valued fuzzy set in a universe $\mathcal{U}$ is thus modelled as an $\mathcal{U} - L^I$ mapping, with $L^I = \{[x_1, x_2] | [x_1, x_2] \subseteq [0, 1]\}$. The lower and upper bound of an element x of L^I will be denoted by x_1 and x_2 respectively: $x = [x_1, x_2]$ (Fig. 1). We will further use the notation $\mathcal{IVFS}(\mathcal{U})$ for the class of interval-valued fuzzy sets over the universe $\mathcal{U}$.

In [15] it is shown that for the partial ordering $\leq_{L^I}$ on L^I given by

$$x \leq_{L^I} y \;\Leftrightarrow\; x_1 \leq y_1 \text{ and } x_2 \leq y_2,\; \forall x, y \in L^I,$$

the structure $(L^I, \leq_{L^I})$ forms a complete lattice (which is a necessary and sufficient condition to do morphology on L^I). The infimum and supremum of an arbitrary subset S of L^I are then respectively given by $\inf S = [\inf_{x\in S} x_1, \inf_{x\in S} x_2]$ and $\sup S = [\sup_{x\in S} x_1, \sup_{x\in S} x_2]$. For $\inf L^I = [0, 0]$ and $\sup L^I = [1, 1]$ we will use the notations 0_{L^I} and 1_{L^I} respectively. Further, the union of an arbitrary family $(A_j)_{j\in J}$ of interval-valued fuzzy sets on $\mathcal{U}$ is defined by $(\bigcup_{j\in J} A_j)(u) = \sup_{j\in J} A_j(u), \forall u \in \mathcal{U}$.

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