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Fuzzy languages with infinite range accepted by fuzzy automata: Pumping Lemma and determinization procedure

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Abstract

The formulation of fuzzy automata allows us to select a great variety of triangular norms. Depending on the selected triangular norm, a fuzzy automaton can accept a fuzzy language (FA-language) with infinite range. These fuzzy automata are not equivalent to the so-called deterministic fuzzy automata (deterministic automata with a fuzzy subset of final states) which only accept fuzzy languages with finite range. In this paper, we study FA-languages with infinite range and a determinization procedure in order to obtain an equivalent fuzzy deterministic automaton for a given fuzzy automaton. A fuzzy deterministic automaton is a fuzzy automaton which satisfies the deterministic condition in its state transition function. The main contributions of our paper are: (1) a Pumping Lemma of FA-languages with infinite range; (2) the formulation of fuzzy deterministic automata and a Pumping Lemma of FDA-languages; (3) the necessary conditions for the determinization of fuzzy automata under continuous triangular norms which accept fuzzy languages of infinite range; and (4) a determinization algorithm for fuzzy automata, its correctness proof and performance.

Keywords: Fuzzy automata; Finite automata; Fuzzy languages; Pumping Lemma; Determinization; Triangular norms

1. Introduction

The theory of fuzzy sets proposed by Zadeh [37] has been widely used for dealing with problems with imprecision and uncertainty. The research on fuzzy languages accepted by fuzzy finite-state machines was originated in the early 1970s by Zadeh [38], Lee and Zadeh [17], and Thomason and Marinos [32]. A fuzzy language L on an alphabet Σ is a fuzzy subset of the free monoid Σ^*, $L : \Sigma^* \to [0, 1]$.

The concept of fuzzy automata was presented by Santos [30] and Wee and Fu [33]. In this paper, we consider automata defined over a finite set of states, i.e., finite-state automata. The formulation of a fuzzy automaton allows it to make a transition from one state to another for a given input symbol to a certain degree. This degree (a truth-value) depends on the previous state degree, the degree of the transition, and the fuzzy operations applied to perform the computation. Thus, the fuzzy language accepted by a fuzzy automaton is constrained by the applied fuzzy operations.

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However, the so-called deterministic fuzzy automata [4] constitutes an exception. A deterministic fuzzy automaton is a simple kind of fuzzy automata: it is a deterministic automaton in which the set of final states is a fuzzy subset of the automaton's states. In a deterministic fuzzy automaton, a word that is accepted by the underlying deterministic automaton, takes the truth-value assigned to its reached final state as acceptance degree. In the following, the fuzzy languages accepted by deterministic fuzzy automata are called DFA-languages.

A fuzzy language $L : \Sigma^* \to [0, 1]$ admits a monoid representation [5]. In that case, $L = \alpha \circ h$ where $\alpha : \Sigma^* \to M$ for a monoid M, and $h : M \to [0, 1]$ is a fuzzy subset of the monoid M. A fuzzy language L is m-recognizable if it has a finite representation (M, h, α). A fuzzy language is m-recognizable if and only if its syntactic monoid M_L is finite [5]. Fuzzy languages accepted by (*max*, *min*)-fuzzy automata, (*max*, *Łukasiewicz intersection*)-fuzzy automata and (*max*, *drastic intersection*)-fuzzy automata are m-recognizable [5]. The basic idea for proving that those languages are m-recognizable consists of (i) defining M_L as the set of all possible fuzzy subsets of states that can be accessible by the fuzzy automaton for every word in Σ^*, and (ii) proving that this set is finite. A similar method was provided in [23] to prove that fuzzy languages accepted by (*max*, *min*)-fuzzy automata are DFA-languages.

Some authors [13,19] have proven that a fuzzy language accepted by a fuzzy automaton with truth-values in a complete residuated lattice $\mathcal{L}$ is a DFA-language if and only if the semiring reduct $\mathcal{L}^*$ of $\mathcal{L}$ is locally finite. Recently, Ignjatović et al. in [14] have shown that DFA-languages are m-recognizable fuzzy languages.

Therefore, designing methods for transforming a fuzzy automaton into its equivalent deterministic fuzzy automaton is a relevant issue. There are several methods for *determinization* of fuzzy automata. The method proposed by Bělohlávek in [4], and Li and Pedrycz in [19] is based on the so-called *powerset construction*, which is used for determinization of ordinary non-deterministic automata [31]. The method proposed by Ignjatović in [13] is a more economical alternative w.r.t. the number of resulting states. This method is based on the so-called *accessible subset construction*, which is analogous to the classic method proposed in [12]. It is worth noting that each deterministic fuzzy automaton is equivalent to a nested (finite) system of deterministic automata [4].

DFA-languages satisfy a Pumping Lemma which is a generalization of the Pumping Lemma of regular languages [31]. Some Pumping Lemmas have been proposed in the literature. These depend on the particular area of fuzzy automata formulation: Malik and Mordeson in [20] provide a Pumping Lemma of (*max*, *min*)-fuzzy languages; Bozapalidis and Louscou-Bozapalidou in [5] introduce a Pumping Lemma of m-recognizable fuzzy languages; Qiu in [28] presents Pumping Lemmas of languages accepted by automata based on complete residuated lattice-valued logic; and Rahonis in [29] introduces a Pumping Lemma of languages accepted by automata based on bounded distributed lattices.

Pumping Lemma of DFA-languages is a necessary condition for a fuzzy language to be a DFA-language. A key property of a DFA-language is that it has finite range. Then, any fuzzy language with infinite range is not a DFA-language and, therefore, it cannot be accepted by any deterministic fuzzy automaton. In general, a (*max*, *triangular norm*)-fuzzy automaton can accept a fuzzy language with infinite range. This kind of fuzzy languages and fuzzy automata are of practical interest in some problems of pattern recognition [3,2]. For example, given a word $\omega \in \Sigma^*$, the fuzzy language $L_\omega : \Sigma^* \to [0, 1]$ such that $L_\omega(\alpha) = 10^{-d(\omega,\alpha)}$ for all $\alpha \in \Sigma^*$ (being $d(\omega, \alpha)$ the Levenshtein distance between the words ω and α), can be interpreted as the vague notion of *the words which are similar to* ω. This language L_ω has infinite range and cannot be accepted by a deterministic fuzzy automaton; however, a (*max*, *product*)-fuzzy automaton can accept it [3]. Fuzzy automata and languages are widely used in lexical analysis, description of natural and programming languages, learning systems, control systems, neural networks, clinical monitoring, pattern recognition, databases, discrete event systems, and many other areas (see [18,23,36,9,27] and references therein).

In this paper, we study fuzzy automata with truth-values in the well-known algebra $\mathcal{A} = ([0, 1], max, min, \otimes, 0, 1)$ where $\otimes$ is a triangular norm (t-norm). Preliminaries to recall definitions and results concerning fuzzy sets and relations are presented in Section 2. In Section 3, we define fuzzy automata (over $\mathcal{A}$) and fuzzy languages accepted by fuzzy automata (FA-languages). In order to simplify some related results, we assume that the t-norm is *without zero divisors*. The formulation to obtain the FA-language accepted by a fuzzy automaton is based on the t-composition of fuzzy relations. As fuzzy automata are an effective generalization of non-deterministic automata, we demonstrate that the support of a FA-language is a regular language (Corollary 1). In Section 4, we show how to define the graph of a fuzzy automaton and to compute the FA-language accepted by the fuzzy automaton based on its graph. Both formulations, based on relations or graphs, are equally valid to calculate FA-languages (Lemma 2). As cycles in the graph of a fuzzy automaton are important for the rest of the paper, we study their main properties in Section 4.1. In general, FA-languages can have infinite range. This happens when there is a cycle in the graph and the t-norm