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The universal fuzzy automaton ☆

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Abstract

In this paper, we introduce the notion of universal fuzzy automaton with membership values in a complete residuated lattice, whose states are the factorizations of this fuzzy language and transition function is defined using the inclusion degree of related fuzzy languages. Next, we define the homomorphism of fuzzy automata, prove that every automaton accepting a fuzzy language can canonically map into the universal fuzzy automaton of this language, which is called the universal property. For a fuzzy language, the connections between the universal fuzzy automaton and fuzzy minimal automata of the given fuzzy language are exploited. Finally, we give a method to construct the universal fuzzy automaton by a deterministic fuzzy automaton accepting the given fuzzy language, which is effective in the case that this deterministic fuzzy automaton is finite.

Keywords: Algebra; Fuzzy relation; Fuzzy automaton; Fuzzy language; Morphism

1. Introduction

The universal automaton of a language has been defined by Conway [1] as a "factor matrix". Another view credits Carrez cames of what seems to be the first definition of the universal automaton in a report that remained unpublished [2,3]. A complete study of properties of this automaton can be found in [4,5]. The universal automaton of a language L contains many interesting information (e.g., factorization) of L and play a very important role in constructing the minimal nondeterministic finite automaton (NFA) of L [6] and regular language learning [7]. Indeed, the universal automaton satisfies the universal property which states that it contains a morphic image of any automaton which recognizes the given language, and thus a copy of any minimal NFA which recognizes the language.

On the other hand, the notion of fuzzy automata was introduced in the very early age of fuzzy set theory [8,9]. Since finite automata constitute a mathematical model of computation, fuzzy finite automata may be considered as an extended model which includes notions like "vagueness" and "imprecision", i.e., notions frequently encountered in the study of natural languages. So investigating fuzzy finite-state automata might reduce the gap between formal languages as studied in classical automata theory [10] on one hand and natural languages on the other hand.

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Usually, fuzzy automata took values in the unit interval [0, 1]. To enhance the processing ability of fuzzy automata, the membership grades were extended to much general algebraic structures. For example, automata theory based on lattice-ordered monoids has been established in [11]. Fuzzy automata are a generalization of NFAs, and the mentioned problems concerning minimal NFA, still exist in work with fuzzy automata. There are many work on the size reduction or minimization of fuzzy automata [9,12–15] based on the idea of computing and merging indistinguishable states, and minimal fuzzy automata by fuzzy equivalence [16,17]. However, except the minimization of deterministic fuzzy automata of a given fuzzy language [15,18], there is no further work to study general problem of the minimal (nondeterministic) fuzzy automata for a given fuzzy language. Since universal automaton is an important tool to study the minimization of NFA, the natural problem is how to define the similar notion for fuzzy automata. This forms the main concern of this paper.

Using the factorizations of a fuzzy language, we shall introduce the notion of universal fuzzy automaton of the given language. The universal property of universal fuzzy automaton is exploited, and the connections of universal fuzzy automaton with the minimization of (nondeterministic) fuzzy automata is discussed. An effective method to construct the universal fuzzy automaton by the deterministic fuzzy automaton accepting the given fuzzy language is presented in this paper.

The construction of universal fuzzy automaton is not a trivial generalization of that of classical universal automaton, its structure depends heavily on the residuated structure of truth values domain. Indeed, we use a special kind of lattice-ordered monoid – complete residuated lattice – as the truth value domain of fuzzy automata instead of lattice-ordered monoid. There are many papers on fuzzy automata with membership values in a complete residuated lattice [16,17,19–21]. As stated in Refs. [16,17,22], the minimization of fuzzy automata heavily depended on the residuated structure of the used complete residuated lattice. To the effective construction of universal fuzzy automaton, we need the notion of deterministic fuzzy automata, which was introduced in Refs. [11,23]. On the determinization of fuzzy automata, we refer to the work of Refs. [15,22].

The content of this paper is arranged as follows. In Section 2, the notion of fuzzy automata with membership values in a complete residuated lattice is recalled. Section 3 introduces factorizations of a fuzzy language. In Section 4, universal fuzzy automaton of a fuzzy language is defined. Section 5 studies the universality of the universal fuzzy automaton. An effective construction of universal fuzzy automaton is given in Section 6. Some remarks are included in the conclusion part.

2. Preliminaries

In this paper we will use complete residuated lattice as the structure of truth value domain of fuzzy logic. A residuated lattice is an algebra $\mathcal{L} = (L, \vee, \wedge, \otimes, \rightarrow, 0, 1)$ such that

(L1) $(L, \vee, \wedge, 0, 1)$ is a lattice with the least element 0 and the greatest element 1,

(L2) $(L, \otimes, 1)$ is a commutative monoid with the unit 1,

(L3) $\otimes$ and $\rightarrow$ form an adjoint pair, i.e., they satisfy the adjunction property: for all $x, y, z \in L$,

$$x \otimes y \leqslant z \quad \Longleftrightarrow \quad x \leqslant y \rightarrow z. \tag{1}$$

If, in addition, $(L, \vee, \wedge, 0, 1)$ is a complete lattice, then $\mathcal{L}$ is called a complete residuated lattice. The operations $\otimes$ (called multiplication) and $\rightarrow$ (called residuum) are intended for modeling the conjunction and implication of the corresponding logical calculus, and supremum ($\bigvee$) and infimum ($\bigwedge$) are intended for modeling the existential and universal quantifier, respectively.

It can be easily verified that with respect to $\leqslant$, $\otimes$ is monotonic in both arguments, $\rightarrow$ is monotonic in the second and anti-monotonic in the first argument, and for any $x, y, z \in L$ and any $\{x_i\}_{i\in I}, \{y_i\}_{i\in I} \subseteq L$, the following hold:

$$\left(\bigvee_{i\in I} x_i\right) \otimes x = \bigvee_{i\in I} (x_i \otimes x), \tag{2}$$

$$\left(\bigvee_{i\in I} x_i\right) \rightarrow x = \bigwedge_{i\in I} (x_i \rightarrow x), \tag{3}$$

$$x \rightarrow \left(\bigwedge_{i\in I} x_i\right) = \bigwedge_{i\in I} (x \rightarrow x_i), \tag{4}$$