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Fuzzy information systems and their homomorphisms

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Abstract

With the arrival of the information age, information acquisition and communication have become more and more important in the field of information technology. This paper uses the concept of homomorphism as a basic tool to study the communication between fuzzy information systems. The concepts of consistent and compatible mappings with respect to fuzzy sets are firstly defined and their basic properties are studied. Then, a pair of lower and upper rough fuzzy approximation operators is constructed by means of the concept of fuzzy mappings. Basic invariant properties of the approximation operators are investigated. Finally, the concepts of fuzzy information system and its homomorphism are introduced, and some invariant properties of fuzzy information systems under homomorphisms are examined. It is proved that the attribute reductions of an original information system and its image system are equivalent to each other in the context of fuzzy attributes. These results may have potential applications in attribute reduction and classification issues.

Keywords: Fuzzy mapping; Rough set; Homomorphism; Attribute reduction

1. Introduction

Information system is a database that describes the relationships between objects and attributes; it is one of the most important mathematical models in the field of artificial intelligence. With the arrival of the information age, access to information, analysis and synthesis of information have become a focus in the field of information technology. Information uncertainty analysis, information fusion, attribute reduction and knowledge classification are becoming more and more important.

Rough set theory [1–3] is a useful tool for dealing with uncertainty, vagueness, and incompleteness of information. It has been applied to rule extraction [4–9], reasoning with uncertainty [10–14], uncertainty modeling [15–22], classification and feature selection [23–29]. In recent years, the communication between information systems has been an important problem in granular computing [30–33]. Many topics on this problem have been widely investigated by

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using fuzzy sets and rough sets [34–41]. One of the main objectives of research on communication between information systems is to reduce large-stored data volume and redundant attributes while preserving some basic information [36–41].

Communication is directly related to the issue of transformations of information systems while preserving their basic properties. As explained in [31–33], communication allows one to translate the information contained in one granular world into the granularity of another granular world and thus provides a mechanism for exchanging information with other granular worlds. From the mathematical viewpoint, this kind of communication can be explained as comparing some structures and properties of different information systems via mappings, which are useful tools to study the relationship between information systems. Thus, the approximations and reductions in an original information system can be regarded as encoding, while the image system is seen as an interpretive information system. The notion of homomorphism based on rough set theory, introduced by Grzymala-Busse [34,35], can be viewed as a special mapping between information systems and used for aggregating sets of objects, attributes, and descriptors of the original system [35–41]. This idea perhaps is a good approach for reducing large-stored data volume and data attributes. In [35], Grzymala-Busse depicted the conditions which make an information system endomorphic. In [36], Deyu Li discussed some invariant properties of discrete information systems under homomorphism. In [37–40], Wang et al. investigated some invariant properties of generalized information systems under homomorphisms and proved that attribute reductions in the original system and image system are equivalent to each other under the condition of homomorphism.

In reality, there are a lot of information systems in which knowledge attributes are vague or ambiguous [42–47]. If each attribute induces a fuzzy set on the sample space, we call this kind of information system fuzzy information system. If all fuzzy sets induced by attributes can define a fuzzy covering on the sample space, the fuzzy information system is called fuzzy covering information system. Fuzzy information systems are extensively used in the fields of fuzzy decision making [42,45,47] and fuzzy comprehensive evaluations [43,44,46]. Since the topic on homomorphism between information systems has become an important problem, how to search for homomorphisms between fuzzy information systems and examine their invariant properties are our new issues.

In this paper, we introduce rough sets as a basic tool to study homomorphisms between fuzzy information systems. By Zadeh's extension principle [48], we define a class of special fuzzy mappings: consistent and compatible mappings and study their basic invariant properties. We then construct a pair of rough fuzzy approximation operators by means of fuzzy mappings and examine their main properties. Finally, we propose the concepts of homomorphism between fuzzy information systems and prove that the attribute reductions of a fuzzy information system and its image system are equivalent to each other under the condition of homomorphism.

The remainder of this paper is organized as follows. In Section 2, we review the relevant concepts in rough sets and fuzzy sets. In Section 3, we study some invariant properties of consistent and compatible mappings and construct a pair of rough fuzzy approximation operators by means of fuzzy mappings. In Section 4, we introduce the concept of homomorphism between fuzzy information systems and we get its main results. Conclusions are drawn in Section 5.

2. Some basic notions of fuzzy and rough sets

We first review some basic concepts related to classical rough sets that can be found in [1–3].

An information system is a pair $IS = (U, C)$, where U is a nonempty finite set of objects and $C = \{a_1, a_2, \ldots, a_m\}$ is a nonempty and finite set of attributes describing objects. For any subset of attributes $B \subseteq C$, we can define an equivalence relation $IND(B)$ as follows.

$$IND(B) = \big\{(x, y) \in U \times U\colon\ a(x) = a(y),\ \forall a \in B\big\}.$$

Obviously, $IND(B) = \bigcap_{a\in B} IND(\{a\})$. By $[x]_B$ we denote the equivalence class of x with respect to $IND(B)$. For any subset $X \subseteq U$, $\underline{B}X = \{x \in U\colon\ [x]_B \subseteq X\}$ and $\overline{B}X = \{x \in U\colon\ [x]_B \cap X \neq \emptyset\}$ are called B-lower and B-upper approximations of X in IS, respectively.

An attribute $a \in B \subseteq C$ is superfluous in B if $IND(B) = IND(B - \{a\})$, otherwise a is indispensable in B. The collection of all indispensable attributes in C is called the core of IS. We say that $B \subseteq C$ is independent in IS if every attribute in B is indispensable in B. $B \subseteq C$ is called a reduct in IS if B is independent and $IND(B) = IND(C)$.

Fuzzy sets are another important extension of classical crisp sets [48,49]. Let U be a universal set. A fuzzy subset A of U is defined as a function assigning to each element x of U a value $A(x) \in [0, 1]$ and $A(x)$ is referred to as the