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A parametric model for determining consensus priority vectors from fuzzy comparison matrices ☆

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Abstract

We consider a group decision-making problem where a set of alternatives have to be ranked according to fuzzy preference judgments given by multiple experts. We assume that expert assessments are expressed in the form of fuzzy multiplicative preference relations or fuzzy comparison matrices. In this paper we propose a general model to generate crisp priority weights of the alternatives from possibly inconsistent and conflicting fuzzy preference relations. We express our model in terms of matrix approximations to address the consistency problem and we use weighted metrics to simulate the group dynamics. Matrix approximation techniques for deriving a common crisp consistent matrix are extended to work with fuzzy matrices by using the concept of α-cuts. In the aggregation process, the importance of each expert is taken into consideration according to the agreement of the group with the expert. This results in a parametric optimization problem for which a computational formulation is given.

Keywords: Decision analysis; Group decision making; Analytic hierarchy process; Fuzzy number; Fuzzy comparison matrix

1. Introduction

The problem of importance weight analysis and determination from multiple source information is a critical issue in many fields like multicriteria decision making, machine learning, meta-search engines, etc. We address this problem in a group decision-making scenario, where a set of alternatives have to be ranked according to their priority weights from preference information provided by a group of decision makers or experts.

We assume each expert expresses his preference judgments of alternatives by means a pairwise comparison matrix [22], which is a common model in this context. These matrices are the basis of the Analytical Hierarchy Process (AHP), a widely used multicriteria decision-making method developed by Saaty [22]. To capture certain complexities

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of real world problems (e.g. uncertainty and, incomplete and vague information), we will work with fuzzy comparison matrices to express imprecision in judgments.

Additionally, in group decision problems, situations of conflict and agreement arise among experts. Our goal is then to derive group priority weights for the alternatives that best reflect conflicting and possibly not consistent preferences given in fuzzy comparison matrices from the experts.

The main approaches in the literature [2,12–14] to address group decision making are the aggregation of individual priorities (AIP approach) and the aggregation of individual judgments (AIJ approach). In the first approach (AIP), the priority vectors are computed from the individual matrices and then, they are aggregated to form the final priority vector. In the AIJ approach, a new group matrix is obtained by aggregating individual matrices. Then a priority vector is derived from this group matrix using a prioritization method. Most of the procedures in this context are articulated in two phases: aggregation of individual preferences and exploitation of collective preference. Our approach integrates the synthesis and the prioritization stages by considering the case where the group wishes to find a compromise consistent matrix that most closely approximates matrices given by the experts. In particular, we will use the general l_p metric as a measure for closeness, this idea was introduced by Yu [29] in the context of group decision problems dealing with utility functions. The problem is then stated as a minimization problem, where the group objective function is defined through the l_p metric and the distances of the individual matrices to the ideal one. This provides a general framework for investigating the consistency problem and the aggregation of individual preferences.

Some methods have been developed in the literature for dealing with fuzzy comparison matrices. Most of them are based on a logarithmic least squares procedure that works with specific discretization of triangular fuzzy data [3,15,16,23,26,27]. In all of these, fuzzy weights are provided, then a defuzzification procedure is needed for deriving the final ranking of the alternatives. In [24,28], goal programming methods are given. In [19,20], triangular membership functions are used to express the expert's satisfaction with different ratios of weights. Then fuzzy preference programming methods that lead to nonlinear optimization problems are developed. A geometric mean method is proposed for trapezoidal fuzzy data in [4].

In this paper, we propose a general model to generate crisp priority weights from possibly inconsistent and conflicting fuzzy comparison matrices. It is articulated in terms of matrix approximation to address the consistency problem and uses a weighted metric to simulate the dynamics of the group. The matrix approximation approach to find a common crisp consistent matrix is extended to deal with fuzzy matrices by working with the concept of α-cuts. In the aggregation process, the importance of each expert is taken into consideration according to the agreement of the group with the expert. This results in a parametric optimization problem for which a computational formulation is given.

The paper is organized as follows. In Section 2, we introduce main definitions and results relating to pairwise comparison matrices and to fuzzy numbers. Section 3 studies the consistency problem dealing with fuzzy comparison matrices. Then, in Section 4, we extend the approximation framework to cover the group problem, where multiple experts provide different and possibly conflicting information. In Section 5, we discuss some conclusions that can be drawn from the proposed model.

2. Preliminaries

This section is devoted to introduce main concepts, results, and notation that we are going to use throughout the paper.

Let $X = \{x_1, x_2, \dots, x_n\}$ $(n \geqslant 2)$ be a finite set of alternatives. We assume that preferences related to these alternatives are formulated in the form of paired comparisons. The estimation m_{ij} expresses the preference intensity of alternative x_i with respect to x_j, where $m_{ij} > 0$. Then an $n \times n$ positive matrix $M = (m_{ij})$ is obtained to dictate the preference judgments given by an expert. Based on the construction of these comparison matrices [22], some properties can be expected.

Definition 2.1. An $n \times n$ positive matrix $M = (m_{ij})$ is said to be *reciprocal* if $m_{ij} = 1/m_{ji}$, $\forall i, j = 1, \dots, n$.

Definition 2.2. An $n \times n$ positive matrix $M = (m_{ij})$ is said to be *consistent* if $m_{ij} \cdot m_{jk} = m_{ik}$, $\forall i, j = 1, \dots, n$.

A reciprocal matrix is named pairwise comparison matrix in the context of AHP. Observe that consistent matrices are reciprocal. A problem of interest is deriving a positive weight vector $w = (w_1, w_2, \dots, w_n)^T$ from a pairwise