Available online at www.sciencedirect.com

ScienceDirect

Fuzzy Sets and Systems 246 (2014) 62–77

FUZZY sets and systems

www.elsevier.com/locate/fss

The ordinal consistency of an incomplete reciprocal preference relation

Yejun Xu [a,b,*], Jatinder N.D. Gupta [c], Huimin Wang [a,b]

[a] *State Key Laboratory of Hydrology-Water Resources and Hydraulic Engineering, Hohai University, Nanjing 210098, PR China*
[b] *Research Institute of Management Science, Business School, Hohai University, Nanjing 211100, PR China*
[c] *College of Business Administration, The University of Alabama in Huntsville, Huntsville, AL 35899, USA*

Received 12 November 2012; received in revised form 1 August 2013; accepted 2 August 2013

Available online 11 August 2013

Abstract

The ordinal consistency is the usual weak transitivity condition that a logical and consistent person should use if he/she does not want to express inconsistent opinions, and therefore becomes the minimum requirement condition that a consistent reciprocal preference relation should verify. In this paper, we define and study the ordinal consistency of an incomplete reciprocal preference relation. We then develop an algorithm to judge whether an incomplete reciprocal preference relation is ordinally consistent. This proposed algorithm can also find all cycles of length 3 to n in the incomplete digraph of the incomplete reciprocal preference relation. Based on this proposed algorithm and two rules, we develop another algorithm to repair an inconsistent incomplete reciprocal preference relation and to convert it to one with ordinal consistency. Our algorithm eliminates the cycles of length 3 to n in the digraph of an incomplete reciprocal preference relation most effectively. Our proposed method can preserve the initial preference information as much as possible. Furthermore, the proposed method can be used for an incomplete reciprocal preference relation with strict comparison and non-strict comparison information. Finally, the effectiveness and validity of the proposed method are illustrated with examples.

Keywords: Ordinal consistency; Incomplete reciprocal preference relation; Cycles

1. Introduction

Decision making procedures, which usually try to find the best alternative(s) from a feasible set, are increasingly being used in various fields for evaluation, selection, and prioritization purposes. In the process of decision making, decision maker (DM) generally needs to compare a set of decision alternatives with respect to a single criterion, and constructs a preference matrix. One of the most popular representations of preference is a reciprocal relation [30] (or called fuzzy preference relation) which has received a great deal of attention from researchers over the last decades. A complete reciprocal preference relation of order n necessitates the completion of all $n(n-1)/2$ judgments in its entire top triangular portion. Sometimes, however, a decision maker (DM) may develop a reciprocal preference

* Corresponding author. Tel.: +86 25 68514612; fax: +86 25 85427972.
E-mail address: xuyejohn@163.com (Y. Xu).

0165-0114/$ – see front matter
http://dx.doi.org/10.1016/j.fss.2013.08.001

relation with incomplete information [20–22,29] because of (1) the time pressure, lack of knowledge, and the DM's limited expertise related to the problem domain [8,24,41,43,45]; (2) the large number of alternatives, n, as it may be practically impossible, or at least unacceptable from the point of view of the decision maker, to perform all the $n(n-1)/2$ required comparisons to complete the pairwise comparison matrices [13]; (3) the convenience/necessity to skip some direct critical comparison between alternatives, even if the total number of alternatives is small [13]; and (4) an expert's inability to efficiently express any kind of preference degree between two or more of the available options. This may be due to an expert not possessing a precise or sufficient level of knowledge of part of the problem, or because that expert is unable to discriminate the degree to which some options are better than others [2,3,16,17]. Existing literature describes theoretical studies related to incomplete reciprocal preference relations.

Transitivity is a fundamental notion in decision theory. It is universally assumed in disciplines of decision theory and is generally accepted in the principle of rationality. Yet, it is often violated in actual choice, particularly in pairwise choices. Therefore, our first task in decision sciences is the resolution of intransitivities [27]. The transitivity assumption can be used to check the judgment consistency of a decision maker (DM). If a DM provides a preference relation that does not possess transitivity (i.e., inconsistency problems exist) [33], the ranking result of alternatives is misleading [18,26,35]. Transitivity of a reciprocal preference relation has received great attention in the past decades [7,10–12,14,34]. There are various types of T-transitivity and stochastic transitivity. Świtalski [34] developed a general class of transitivity conditions (FG-transitivity) for reciprocal preference relations. De Baets et al. [12] presented the cycle-transitivity framework for studying the transitivity of reciprocal relations. De Baets and Meyer [11] discussed and compared FG-transitivity framework and cycle-transitivity framework. Their study turns out that FG-transitivity is less elegant when expressed in the cycle-transitivity framework, and cycle-transitivity has proven to be more general.

The consistency in preference relations (whether multiplicative and reciprocal) given by DMs has a direct impact on the ranking results of the final decision. The consistency was first addressed by Saaty in AHP [32], and later was extended to the reciprocal preference relations [18]. The consistency of incomplete reciprocal preference relations have received attention only recently [2,3,13,17,25,38,40]. In practical decision-making, perfect multiplicative or additive consistency is difficult to obtain, because people's judgment rarely conforms to an exact mathematical formula. Therefore, the consistency level is checked by a consistency index CI. If a reciprocal preference relation does not satisfy the consistency level, a few methods are proposed to repair the inconsistency of reciprocal preference relations. Ma et al. [26] proposed an algorithm to repair an inconsistent reciprocal preference relation to convert it to weak transitivity. Xia et al. [37] developed an algorithm to improve the consistency level of a reciprocal [0, 1]-valued preference relation. They developed the corresponding algorithm for the incomplete reciprocal [0, 1]-valued preference relation. However, Kwiesielewicz and van Uden [23] showed that even if a matrix successfully passes a test in the sense of the analytical hierarchy process, it can be contradictory. The ordinal consistency is the usual weak transitivity condition that a logical and consistent person should use if he/she does not want to express inconsistent opinions. Therefore, it is the minimum required condition that a consistent reciprocal preference relation should satisfy [36].

However, little work has been done on the ordinal consistency problem in incomplete reciprocal preference relations. In addition, lack of consistency in decision making often leads to inconsistent conclusions, why it is important, if not crucial, to study conditions under which consistent preference relations can be obtained. We will propose a new method to solve the inconsistent problem for the incomplete reciprocal preference relations. This is the subject of the present paper.

In this paper, we propose a novel method for decision maker to repair the inconsistency of a class of incomplete reciprocal preference relations. The proposed method consists of two processes: (1) judgment process on ordinal consistency and inconsistency, and (2) inconsistency repairing process. The former process is to find all the unreasonable cycles of length 3 to n in the digraph of an incomplete reciprocal preference relation. The second process deals with efficiently repairing the inconsistency of an incomplete reciprocal preference relation to reaching ordinal consistency while preserving the original preference information of DM. In this process, two rules are stipulated to help the decision maker for modification. These two processes are realized using two proposed algorithms.

The rest of the paper is organized as follows. Section 2 gives the basic concepts of the complete reciprocal preference relation and incomplete reciprocal relation. We give the definition of ordinal consistency of an incomplete reciprocal preference relation and an adjacency matrix of an incomplete reciprocal preference relation. In Section 3, an algorithm is developed to judge whether an incomplete reciprocal preference relation is ordinally consistent, as well as to find all cycles in the digraph of an incomplete reciprocal preference relation. A method for repairing the