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Compromise principle based methods of identifying capacities in the framework of multicriteria decision analysis

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Abstract

The basic aim of the compromise principle employed in this paper is to seek the capacity by which each alternative has a relatively equal chance to reach as close as possible to its highest reachable overall evaluation. According to the compromise principle, three types of capacity identification methods – the simple arithmetic average based compromise method, the least squares based compromise method and the linear programming based compromise method – are proposed. The input information required for the compromise principle based identification methods consists of the preference information with respect to the decision criteria and the partial evaluations of the alternatives provided by the decision maker. An illustrative example is given to show the processes of the proposed methods, and a comparison analysis with the maximum entropy principle based identification method is also presented.

Keywords: Decision analysis; Capacity; Choquet integral; Compromise principle; Entropy

1. Introduction

In the framework of multicriteria decision analysis, the capacities [6] (or called nonadditive measures [7], fuzzy measures [32]), normal monotone set functions on the criteria set, can adequately describe the importance of each criterion as well as the interaction among the criteria by evaluating the weight of any combination of criteria [11,14]. However, this exponential complexity of its construction [15] restricts the applicability of the capacity based multicriteria decision models [11,12].

In order to reduce the number of parameters required, and then reduce the complexity of weight evaluation process, some particular families of capacities have been proposed, such as the λ-capacities [32], the possibility capacities [37], k-additive capacities [13], p-symmetric capacities [30], and the k-tolerant and k-intolerant capacities [25].

In practice, we usually need to construct some optimization models to help identify the capacities or the particular families on criteria set. In 2008, Grabisch, Kojadinovic and Meyer [14] provided a comprehensive overview on the main approaches of identifying capacities. Mainly according to the type of the objective function, these methods

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can be classified into four major categories [14,15]: the least squares based approaches [16], the maximum split approaches [27], the TOMASO (Tool for Ordinal Multi-Attribute Sorting and Ordering) approaches [29,31], and the maximum entropy principle based approaches [22–24]. Lately, Beliakov [3] adopted the least absolute deviation criterion and constructed a linear programming model to identify capacities. The method proposed by Beliakov [3] is relatively easy to solve and also applicable for the interval cases. Angilella, Greco and Matarazzo [1] developed the nonadditive robust ordinal regression model to identify capacities. Fallah Tehrani et al. [9] proposed the maximum log-likelihood principle based optimization model to identify the capacities in the context of yes/no decision or binary classification. Hüllermeier and Fallah Tehrani [20] further gave two efficient approaches to realize the maximum log-likelihood principle based optimization model under the 2-additive cases. Fallah Tehrani et al. [10] proposed the maximum margin principle based optimization model to identify the capacities with the ordered categorical preference information. An indispensable component of most of the above approaches (except for the maximum entropy principle based approaches) is the learning set [14], which consists of some decision alternatives and the decision maker's preference of them. That preference may be a partial weak order on the alternatives in some approaches, e.g., the maximum split approaches. However, in most of the approaches, the preference should be the desired overall evaluations of all alternatives in the learning set. It is indeed a very time-consuming process for the decision maker to provide such preference [14]. Just as Grabisch and Labreuche written in literature [15], such a requirement "is somewhat odd in decision theory", usually the desired overall evaluation is not known, and these approaches are "much more related to the field of estimation theory".

On the other hand, some works tried to derive capacities only from the decision maker's preference with respect to decision criteria (importance of criteria, interaction among them, etc.). Tagahagi [33] introduced an identification method by using the diamond pairwise comparison and the phi(s) conversation. Wu and Zhang [34] proposed an identification method of 2-additive capacities based on the diamond pairwise comparison and the maximum entropy principle. These two methods have a common source of the decision maker's preference information, i.e., the diamond pairwise comparison, in which the decision maker should provide the relative importance and interaction for each pair of criteria. Generally speaking, it is rather difficult for the decision maker to set a definite value to the importance of each decision criterion as well as the interaction among them, even when the criteria set only consists of two criteria.

However, the decision maker can easily express his/her preference with respect to the criteria like the following [1,2,14,15,22,27,28,31]: "the criterion i is more important than the criterion j", "the interaction between the criteria i and j is significant (or slightly) positive (or negative)", "the interaction between the criteria i and j is larger than that between the criteria k and l", etc. Obviously, such preference information can only constitute a region of the feasible capacities. Some additional selection principles (e.g., the maximum entropy principle) or constraint conditions should be employed to identify the most desired capacity(ies). Furthermore, the maximum entropy principle based methods, also the methods in literatures [33,34], more or less neglect the information about the alternatives provided by the decision makers, e.g., the partial evaluations of the alternatives.

In this paper, we adopt a compromise principle to get the most desired capacity(ies) from the set of the feasible capacities. The compromise principle means that the desired capacity should give each decision alternative an equal chance to reach as close as possible to its highest reachable overall evaluations. We give three types of methods to apply the compromise principle, i.e., the simple arithmetic average based compromise method, the least squares based compromise method and the linear programming based compromise method. The first method takes the simple arithmetic average of the capacities that enable the alternatives to reach their highest overall evaluations as the final desired capacity. The second and third methods seek the capacity(ies) that hold(s) the minimum squared deviation and the minimum absolute deviation between the final overall evaluations and the highest reachable overall evaluations of all alternatives, respectively. The main common characteristic of the proposed methods is that they utilize the preference information with respect to the decision criteria as well as the partial evaluations of the alternatives.

This paper is organized as follows. After the introduction, we introduce some knowledge about the capacity, the Möbius representation, the Shapley interaction index, the entropy of capacity in Section 2. Then, the three types of compromise principle based methods of identifying capacities are proposed in Section 3. Section 4 shows an illustrative example of the proposed methods. Finally, we conclude the paper in Section 5.

For convenience, let $N = \{1, 2, \dots, n\}$ be the criteria set of n criteria and $\mathcal{P}(N)$ be the power set of N.