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A new fuzzy programming approach for multi-period portfolio optimization with return demand and risk control

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Abstract

This paper considers a multi-period portfolio selection problem imposed by return demand and risk control in a fuzzy investment environment, in which the returns of assets are characterized by fuzzy numbers. A fuzzy lower semi-deviation is originally defined as the risk control of portfolio. A proportion entropy constraint is added as the divergence measure of portfolio. Based on the theories of possibility and necessity measures, a new multi-period portfolio optimization model with return demand and risk control is proposed. And then, the proposed model is transformed into a crisp nonlinear programming problem by using fuzzy programming approach. Furthermore, an improved differential evolution algorithm is designed for obtaining the optimal strategy. Finally, a numerical example is given to illustrate the practicality and efficiency of the proposed model and the corresponding algorithm.

Keywords: Finance; Multi-period portfolio selection; Fuzzy mathematical programming; Possibility measure; Necessity measure; Differential evolution algorithm

1. Introduction

Portfolio selection deals with the problem how to allocate investor's wealth among different assets such that the investment goal can be achieved. Markowitz [23] originally proposed the mean-variance (MV) model for portfolio selection in 1952, which has played an important role in the development of modern portfolio selection theory. The main goal is to maximize the expected return for a given level of risk or minimize the expected risk for a given level of expected return. It is a single period model which makes an one-off decision at the beginning of the period and holds on until the end of the period. However, in practical investment, investors often need to adjust their wealth in several consecutive periods. Hence, it is nature to extend the single period portfolio selection to multi-period portfolio selections. So far, the multi-period portfolio selection problems have been investigated by many researchers using different approaches. Mossin [24] presented an optimal multi-period portfolio polices by using dynamic programming approach. Wu and Li [33] investigated dynamic mean-variance portfolio selection with both regime switching and uncertain exit time. Pınar [27] developed and tested under simulated market conditions, robust multi-period (two- and three-stage)

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portfolio selection models based on penalizing a downside-risk term while maximizing the expected end-of-horizon portfolio value. Costa and Araujo [7] explored a discrete-time generalized mean-variance portfolio selection problem subject to Markovian jumps in the parameters. Bäuerle and Rieder [1] investigated the problem of maximizing the expected utility from terminal wealth and solved it by stochastic control methods for different utility functions. Yin and Zhou [36] studied a class of dynamic Markowitz's mean-variance portfolio selection problems and proposed a discrete-time Markov modulated portfolio selection model by taking into consideration of market trend and other factors. Li and Ng [21] considered the mean-variance formulation in multi-period portfolio selection and determined the optimal portfolio policy and an analytical expression of the mean-variance efficient frontier. The same approach is used by Li et al. [20] in a multi-period safety-first formulation. Zhu et al. [44] took a bankruptcy constraint into consideration and obtained an optimal portfolio policy. Zhou and Li [43] investigated a dynamic continuous-time mean-variance portfolio selection problem in the spirit of Markowitz's original work. Yan et al. [35] formulated a class of multi-period semi-variance model. Yan and Li [34] proposed a class of multi-period semi-variance portfolio selection with a four-factor futures price model in stochastic market. Geyer et al. [12] used stochastic linear programming approach to study the multi-period portfolio optimization. Calafiore [4] proposed a model in which periodic optimal portfolio adjustments were determined with the objective of minimizing a cumulative risk measure over the investment horizon, while satisfying portfolio diversity constraints at each period and achieving or exceeding a desired terminal expected wealth target. Wei and Ye [32] considered a multi-period mean-variance portfolio selection model which imposed by a bankruptcy constraint in a stochastic market. Gülpınar and Rustem [14] proposed multiple alternative return and risk scenarios and designed a min-max algorithm to determine an optimal worst-case investment strategy. Bertsimas and Pachamanova [2] presented different robust formulations for the multi-period portfolio optimization problem. Dantzig and Infanger [8] demonstrated how multi-period portfolio optimization problems can be efficiently solved as multi-stage stochastic linear programs. Çakmak and Özekici [3] considered a multi-period mean-variance model where the model parameters change according to a stochastic market. Yu et al. [37] studied the multi-period portfolio selection problem in a financial market using a mini-max principle.

As the above literatures mentioned, they often characterized a financial asset as a random variable with a probability distribution over its return. But there are many non-probabilistic factors that affect the financial markets such that the return of risky asset is fuzzy uncertainty. With the extensive application of fuzzy set theory, some researchers have investigated the portfolio selection problem in fuzzy environment. For example, Inuiguchi and Ramík [17], Inuiguchi and Tanino [18] presented the mini-max regret model to handle possibilistic portfolio selection problem. Li and Xu [22] used modality approach and goal attainment approach to deal with a possibilistic portfolio selection problem with interval center values. As pointed out by Wang and Zhu [31], using fuzzy programming approaches, quantitative analysis, qualitative analysis, the experts' knowledge and the investors' subjective opinions of stock markets can be better integrated in a portfolio selection model to express the variations of stock markets more efficiently. So it is reasonable to say that using fuzzy programming approaches to deal with portfolio selection problem are useful in real investment world. Recently, more and more fuzzy portfolio selection models have been proposed, see for example Hasuike and Ishii [15], Hasuike et al. [16], Sadjadi et al. [28], Zhang et al. [41,42].

Though great progress has been made in applying fuzzy programming approaches to deal with portfolio selection problem in the previous studies, to our knowledge, very few literatures are available in applying fuzzy programming approaches to deal with multi-period portfolio selection problem. Therefore, this paper aims to employ fuzzy programming approach to deal with multi-period portfolio selection problem. The main contributions of our paper can be summarized as follows. We originally propose a risk measure of portfolio by using a fuzzy lower semi-deviation. Based on this risk measure, we construct a multi-period portfolio optimization model with return demand and risk control, which can be better to reflect the reality of the portfolio selection in fuzzy environment. We add the proportion entropy in Kapur [19] into our model to measure the diversification of portfolio. We design an improve differential evolution algorithm for obtaining the optimal strategy.

This paper is organized as follows. In Section 2, we introduce some basic concepts and operations of fuzzy numbers. In Section 3, we define a new fuzzy lower semi-deviation to measure the risk of portfolio. In Section 4, we propose a multi-period portfolio optimization model with return demand and risk control under a fuzzy environment and further convert it into a crisp form by using fuzzy programming approach. In Section 5, an improved differential evolution algorithm is given for obtaining optimal strategy. In Section 6, a numerical example is given to illustrate the idea of the proposed model and some comparative results are given to demonstrate the effectiveness of the proposed fuzzy lower semi-deviation as a risk measure. Finally, we conclude the paper in Section 7.