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A bipolar approach in fuzzy multi-objective linear programming

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Abstract

The traditional frameworks for fuzzy linear optimization problems are inspired by the max–min model proposed by Zimmermann using the Bellman–Zadeh extension principle to aggregate all the fuzzy sets representing flexible (fuzzy) constraints and objective functions together. In this paper, we propose an alternative approach to model fuzzy multi-objective linear programming problems (FMOLPPs) from a perspective of bipolar view in preference modeling. Bipolarity allows us to distinguish between the negative and the positive preferences. Negative preferences denote what is unacceptable while positive preferences are less restrictive and express what is desirable. This framework facilitate a natural fusion of bipolarity in FMOLPPs. The flexible constraints in a fuzzy multi-objective linear programming problem (FMOLPP) are viewed as negative preferences for describing what is somewhat tolerable while the objective functions of the problem are viewed as positive preferences for depicting satisfaction to what is desirable. This approach enables us to handle fuzzy sets representing constraints and objective functions separately and combine them in distinct ways. After aggregating these fuzzy sets separately, coherence (or consistency) condition is used to define the fuzzy decision set.

Keywords: Fuzzy mathematical programming; Fuzzy multi-objective linear programming; Bipolarity; Coherence condition; Aggregation operator; OWA operator

1. Introduction

One of the early models of fuzzy linear programming was proposed by Zimmermann [46,47]. In the model he studied, an aspiration level for the objective function had been assumed to be prescribed along with certain tolerances, and the inequalities in linear constraints were made flexible. Thereafter, the objective functions and the flexible constraints received the same treatment leading to a symmetric model. The classical Bellman–Zadeh principle [4] had been used to define a fuzzy decision set of solutions by aggregating all fuzzy inequalities using the '*min*' aggregation. An optimal solution of the linear program was the one for which this minimum (aggregated function) is maximal. The core ideas of Zimmerman and Bellman–Zadeh paradigms have been adopted in the years to follow so much so that several other fuzzy optimization models ultimately boil down to max–min optimization models. A large literature is available in this context; just to cite some, please refer to [2,3,25,27–29,33,49] and references therein. In [6,20,21,30,36,37,45,47], to name a few, researchers had used various approaches to model FMOLPPs. In particular [6,30,36,47]

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used the max–min approach to solve FMOLPPs. There are certain non-symmetric models for FMOLPP, for instance [7,38]. However, ultimately these models reduce to symmetric model where in the fuzzy sets representing objective functions and the constraints are aggregated using '*min*' aggregation operator.

The main advantage of the max–min framework is that it enables a better discrimination between good and less good solutions to a set described by linear inequalities representing both the fuzzy objective function and the flexible constraints. However, an interpretation of a decision as the intersection of fuzzy sets, computed by applying '*min*' operator to the membership functions of the fuzzy sets of objectives and constraints, implies that there is no compensation between low and high degree of membership. However, an aggregation of subjective categories in the framework of human decisions almost always shows some degree of compensation [48], hence the max–min approach does not found favor in certain situations in decision making problems. With this motivation, Luhandjula [23], Sommer and Pollatschek [31], Tsai et al. [34], Süer et al. [32], Werner [39], proposed and used various compensatory aggregation operators to solve FMOLPPs.

Any approach which aggregates all fuzzy sets representing the objective functions and the constraints together abolishes the distinction between the two. Physically, a constraint is something which should be satisfied, at least to some extent for a flexible constraint. In other words, constraints are something to be respected. On the other hand, there is no idea of requirement associated with the objective functions. The objective functions aspiration levels in FMOLPP only depict the desire or wish of the decision maker (DM) and hence non-compulsory. It makes sense that if a solution of FMOLPP has positive membership degrees in not all but some fuzzy sets representing the objective functions then it should continue to have a positive membership degree on their aggregation. What we observe that the '*min*' aggregation operator, being non-compensatory, does not support this property.

At this point we would like to cite Benferhat et al. [5] and Dubois and Prade [12,13], who suggested the concept of negative preferences and positive preferences to respectively discriminate between what is unacceptable and what is really satisfactory for DM. The negative preferences act as constraints discarding the unacceptable solutions and the positive preferences lead to support appealing or desired solutions. Thus, an FMOLPP can be modeled for computing the best solutions after merging the negative and the positive preferences separately. But this amalgamate of two different kinds of information may create inconsistency. The latter can be enforced by restricting what is desirable to what is tolerated using *coherence or consistency condition* [5]. The above ideas naturally provide a bipolar view of preferences in fuzzy optimization.

Motivated by these thoughts, in this paper, we attempt to study FMOLPPs with afore described bipolar approach. We shall be using separate aggregation schemes for aggregating the flexible constraints with predefined admissible tolerances and the objective functions with preset desired aspiration levels by the DM.

The paper is structured as follows. In Section 2, we present a set of concepts from bipolarity and ordered weighted averaging (OWA) operator which facilitate the subsequent discussion. In Section 3, a general framework for FMOLPPs in a bipolar setting is explained. Section 4 discusses the crisp formulations of FMOLPP using OWA operator and some other compensatory aggregation operators. Section 5 presents a numerical illustration while the paper concludes in Section 6 with some remarks and future directions.

2. Preliminaries

2.1. Bipolarity

Three forms of bipolarity are described in literature, called types I, II, III, for simplicity. The details can be found in [12] and [5]. We briefly recall them here.

Type I: **Symmetric bipolarity.** It relies on the use of single bipolar scale [0, 1] whose highest value 1 means totally sure and lowest value 0 denotes impossible. The neutral value is taken as 0.5 and it refers to the total uncertainty about whether an event or its contrary occurs. Probability measure is one such well-known example. Tversky–Kahneman's [35] cumulative prospect theory uses the entire real line as a bipolar scale. It is important to point out that homogeneity in the information semantic is subsumed in this kind of bipolarity.

Type II: **Homogeneous bivariate bipolarity.** It works with two separate positive and negative scales related via a duality relation. Here, an item is judged according to two independent evaluations: a positive scale and a negative scale. A positive scale (in favor) is denoted by $L^+ = (0, 1]$ where its top represents maximal possible satisfaction and