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Fuzzy nonlinear programming approach for evaluating and ranking process yields with imprecise data

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Abstract

Process yield, the percentage of processed product units passing inspection, is a standard numerical measure of process performance in the manufacturing industry. On the basis of the process yield expression, an index S_{pk} was developed to provide an exact measure of process yield for normally distributed processes. Most traditional studies measuring process capability are based on crisp estimates in which the output process measurements are precise. However, it is common that the measurements of process quality characteristics are insufficiently precise. Traditional approaches for evaluating process yield become unreliable in such cases. Therefore, this study formulates fuzzy numbers to describe the quality characteristic measurements and applies two methods to construct the fuzzy estimation for S_{pk}. A nonlinear programming approach is provided to solve the α-level sets of the estimator, and a testing procedure is presented for making decisions. Finally, this concept is illustrated with an example and extended to solve the ranking problem of multiple yield indices.

Keywords: Fuzzy ranking; Nonlinear programming; Imprecise; Process yield

1. Introduction

Process yield, which is the percentage of processed product units passing inspection, is a standard numerical measure of process performance in the manufacturing industry. For a product to pass inspection, its characteristics must fall within the manufacturing tolerance; all product units that pass inspection are equally acceptable to the producer. A product that is rejected because of failure to conform to the tolerance may be scrapped or may require costly repairs. For processes with two-sided manufacturing specifications, process yield may be calculated as $F(USL) - F(LSL)$, where USL and LSL denote the upper and lower specification limits, respectively, and $F(\cdot)$ represents the cumulative distribution function (CDF) of the process characteristic. If the process characteristic is normally distributed, process yield may be expressed as $\Phi[(USL-\mu)/\sigma] - \Phi[(\mu - LSL)/\sigma]$, where μ denotes the process mean, σ represents the process standard deviation, and $\Phi(\cdot)$ is the CDF of the standard normal distribution.

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Statistical process control (SPC) methods are the basic statistical techniques for quality control and quality assurance. Control charts can indicate the presence of special causes that upset the processes and help engineers to detect, diagnose, and correct production problems in a timely manner. A successful SPC program reduces scrap and rework; in addition, it maintains control of the process and provides information about process stability over time. However, process stability does not ensure that the process meets the product or process specifications. Hence, process capability analysis, which addresses process results with regard to product specifications, is critical to measure process performance. The relationship between the actual process performance and specification limits of tolerance may be quantified using appropriate process capability indices (PCIs). By analyzing PCIs, a production department can trace and improve an inferior process to enhance the quality and satisfy the requirements of customers. The indices C_p, C_a, and C_{pk} are defined explicitly as follows:

$$C_p = \frac{USL - LSL}{6\sigma} = \frac{d}{3\sigma}, \tag{1}$$

$$C_a = 1 - \frac{|\mu - M|}{d}, \tag{2}$$

$$C_{pk} = \min\left\{\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right\} = \frac{d - |\mu - M|}{3\sigma}, \tag{3}$$

where $d = (USL - LSL)/2$ is half the length of the specification interval and $M = (USL + LSL)/2$ is the midpoint of the specification limits. The index C_p measures the overall process variation relative to the specification tolerance. Therefore, C_p only reflects the potential of a process to produce an acceptable product and does not consider where the process is centered. The index C_a measures the degree of process centering, which alerts the user if the process mean deviates from the desired midpoint. Therefore, C_a only reflects process accuracy. The index C_{pk} considers the magnitudes of process variation and degree of process centering, therefore measuring the actual process performance. Given a fixed value of C_{pk}, the bounds on process yield may be obtained as $2\Phi(3C_{pk}) - 1 \leqslant \%\text{Yield} < \Phi(3C_{pk})$ [1]. However, C_{pk} only provides an approximate, rather than an exact, measure of process yield.

To obtain an exact measure of process yield, Boyles [4] presented a PCI S_{pk}, which is defined as

$$\begin{aligned} S_{pk} &= \frac{1}{3}\Phi^{-1}\left\{\frac{1}{2}\Phi\left(\frac{USL - \mu}{\sigma}\right) + \frac{1}{2}\Phi\left(\frac{\mu - LSL}{\sigma}\right)\right\} \\ &= \frac{1}{3}\Phi^{-1}\left\{\frac{1}{2}\Phi[3C_pC_a] + \frac{1}{2}\Phi\left[3C_p(2 - C_a)\right]\right\}, \end{aligned} \tag{4}$$

where $\Phi^{-1}(\cdot)$ denotes the inverse function of $\Phi(\cdot)$. A precise relationship between S_{pk} and process yield may be obtained as $\%\text{Yield} = 2\Phi(3S_{pk}) - 1$. Because S_{pk} provides an exact measure of process yield, it is widely accepted by many engineers and shop floor controllers as a communication tool for evaluating and improving manufacturing quality [13,26].

Traditionally, statistical testing with the null hypothesis H_0: $S_{pk} \leqslant c$ and alternative hypothesis H_1: $S_{pk} > c$, where c is the minimum requirement of process yield, is often used to test process yield. By using sampling data, the decision of whether to reject H_0 can be determined by comparing the estimate of S_{pk} and the critical value c_0. However, it is common that the measurements of process characteristics are insufficiently precise. Fuzziness often arises in real applications from sources including observations with coarse scales, measurement errors that are not quantified accurately, output measurements judging with humans' partial knowledge or subjectivity, the output reading by a digital measurement equipment, but the number of decimals is finite, etc. Therefore, statistical inference in evaluating process yield based on imprecise data may lead to unreliable decision results. A new trend of combining randomness and fuzziness in assessing process capability has recently occurred. Cen [10] proposed the concept of fuzzy quality, based on fuzzy probability, and applied it to compute a PCI C_p. Moreover, Lee [17] and Hong [19] presented the estimator for C_{pk} using fuzzy numbers when the measurement refers to the subjective determination of the decision makers. In addition, Chen et al. [12] provided fuzzy inference to determine the uncertainty in evaluating process performance for processes with one-sided specifications. Tsai and Chen [30] considered C_p in a fuzzy environment and formulated a pair of nonlinear functions to obtain its membership function. In addition, Parchami et al. [27] and Parchami and Mashinchi [28] regarded the specification limits as fuzzy numbers, and subsequently applied the extension principle and the estimation technology of [5,6] to construct fuzzy confidence intervals and fuzzy point