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Discussion

Comment on "Fuzzy mathematical programming for multi objective linear fractional programming problem"

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Abstract

Chakraborty and Gupta, in their paper "Fuzzy mathematical programming for multi objective linear fractional programming problem", published in Fuzzy Sets and Systems 125 (2002), claimed that their methodology proposed for solving multi objective linear fractional programming problem always yields an efficient solution. This paper indicates that their claim is generally wrong and this is due to the non-equivalence of the original problem and the associated linear problem.

Keywords: Fuzzy mathematical programming; Linear fractional programming; Efficient point

The general form of a multi objective linear fractional programming problem (MOLFPP) is

$$\max_{x\in X}\left[Z_1(x), Z_2(x), \dots, Z_k(x)\right],$$

where $X = \{x \in R^n \mid Ax \leqslant b,\ x \geqslant 0\}$ with $b \in R^m$, $A \in R^{m\times n}$ and $Z_i(x) = \frac{c_i x+\alpha_i}{d_i x+\beta_i} = \frac{N_i(x)}{D_i(x)}$ with $c_i, d_i \in R^n$, $\alpha_i, \beta_i \in R$, for each $i = 1, 2, \dots, k$. Without any loss of generality it can be assumed that $d_i x + \beta_i > 0$ for each $i = 1, 2, \dots, k$ and $x \in X$.

Chakraborty and Gupta [1] split objective functions in two categories according to the signs of their nominators and denoted

$$I = \left\{i \in \overline{1,k} \mid N_i(x) \geqslant 0 \text{ for some } x \in X\right\},$$
$$I^c = \left\{i \in \overline{1,k} \mid N_i(x) < 0 \text{ for each } x \in X\right\}.$$

For simplicity, they let t be the least value of $1/D_i(x)$ for $i \in I$ and $-1/N_i(x)$ for $i \in I^c$, i.e.

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$$t = \min\left\{\min_{i\in I}\frac{1}{D_i(x)}, \min_{i\in I^c}\frac{-1}{N_i(x)}\right\}. \tag{1}$$

Then, they replaced (1) by the constraints

$$\begin{aligned} &1/D_i(x) \geqslant t, \quad i \in I,\\ &-1/N_i(x) \geqslant t, \quad i \in I^c. \end{aligned} \tag{2}$$

Note that (1) and (2) are not equivalent: for each $x \in X$, there are many values of t satisfying (2), but there is only one value of t defined in (1). With the help of the transformation $y = tx$ ($t > 0$), they formulated the following multi objective linear programming problem (MOLPP)

$$\begin{aligned} \max\quad & \{g_i(y,t),\ i \in I \cup I^c\}\\ \text{s.t.}\quad & -tN_i(y/t) \leqslant 1, \quad i \in I^c,\\ & tD_i(y/t) \leqslant 1, \quad i \in I,\\ & A(y/t) - b \leqslant 0,\\ & t \geqslant 0, \quad y \geqslant 0, \end{aligned}$$

where

$$g_i(y,t) = \begin{cases} tN_i(y/t), & i \in I\\ tD_i(y/t), & i \in I^c. \end{cases}$$

Further on, they proposed two types of membership functions for objectives from class I and I^c, respectively and constructed a fuzzy programming model associated to MOLPP. Using Zadeh's min operator to translate the meaning of the connective "and", they transformed the fuzzy model to a crisp linear model. Solving the crisp model they obtained an efficient solution (y^*, t^*) to MOLPP. Finally, they derived a solution $x^* = y^*/t^*$ to MOLFPP. Essentially based on the facts formulated and proved for single objective fractional programming problems [1, Definition 1, Theorems 2 and 3], Chakraborty and Gupta claimed that MOLFPP and MOLPP are equivalent, and the efficient solution (y^*, t^*) to MOLPP yields the efficient solution x^* to MOLFPP. Ironically, for both examples they solved in [1], they obtained non-efficient solutions.

Example 1. Solving

$$\begin{aligned} \max\quad & \left(z_1(x) = \frac{-3x_1 + 2x_2}{x_1 + x_2 + 3}, z_2(x) = \frac{7x_1 + x_2}{5x_1 + 2x_2 + 1}\right)\\ \text{s.t.}\quad & x_1 - x_2 \geqslant 1\\ & 2x_1 + 3x_2 \leqslant 15,\\ & x_1 \geqslant 3,\\ & x_1, x_2 \geqslant 0, \end{aligned} \tag{3}$$

the solution obtained by Chakraborty and Gupta was $x^* = (3, 2)$ that is not efficient as they claimed. For this solution, $z_1^* = -0.625$, $z_2^* = 1.15$, while for $x = (3.6288, 2.5808)$, $z_1(x) = -0.6216$ and $z_2(x) = 1.1513$. Thus, both objective functions have larger values at the feasible point x than at x^*.

Example 2. Solving

$$\max\quad \left(z_1(x) = \frac{-3x_1 + 2x_2}{x_1 + x_2 + 3}, z_2(x) = \frac{7x_1 + x_2}{5x_1 + 2x_2 + 1}, z_3(x) = \frac{x_1 + 4x_2}{2x_1 + 3x_2 + 2}\right) \tag{4}$$

subject to the same constraints as in the previous example, the solution obtained by Chakraborty and Gupta was $x^* = (3, 2)$ that is again not efficient. For this solution, $z_1^* = -0.625$, $z_2^* = 1.15$, $z_3^* = 0.7857$, while for $x = (3.6288, 2.5808)$, $z_1(x) = -0.6216$, $z_2(x) = 1.1513$ and $z_3(x) = 0.8207$. Thus, all three objective functions have larger values at the feasible point x than at x^*.