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Fuzzy Sets and Systems 241 (2014) 76–93

FUZZY sets and systems

www.elsevier.com/locate/fss

Aggregation functions on the set of discrete fuzzy numbers defined from a pair of discrete aggregations

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Available online 23 September 2013

Abstract

In this paper we propose a method to construct aggregation functions on the set of discrete fuzzy numbers whose support is a set of consecutive natural numbers from a couple of discrete aggregation functions. The interest on these discrete fuzzy numbers lies on the fact that they can be interpreted as linguistic expert valuations that increase the flexibility of the elicitation of qualitative information based on linguistic terms. Finally, a linguistic decision making model based on a pair of aggregation functions defined on discrete fuzzy numbers is given.

Keywords: Fuzzy connectives and aggregation operators; Decision making; Linguistic variable; Linguistic term; Discrete fuzzy number; Discrete aggregation function; Uninorm; Nullnorm

1. Introduction

The aggregation of information, understood as a process of merging all collected data into a concrete representative value, has become a field of increasing interest because of the great quantity of applications which include many subjects not only from mathematics and computer sciences, but also from many applied fields like economics and social sciences (see for instance [1,12]). The process of fusioning some data into a representative output is usually carried out by the so-called aggregation functions that have been extensively investigated in the last few decades [1,2,28]. Decision making, subjective evaluations, optimization and control are, among others, examples of concrete application fields where aggregation functions become an essential tool. In all these fields, it is well known that the data to be aggregated vary among many different kinds of information, from quantitative to qualitative information. Moreover, many times some uncertainty is inherent to such information.

Many different tools have been proposed for managing uncertainty, specially fuzzy logic and fuzzy sets. In this sense, some generalizations and extensions of fuzzy sets have also been considered like interval-valued fuzzy sets, intuitionistic fuzzy sets, type-2 fuzzy sets, multidimensional fuzzy sets. Recently, also *Hesitant fuzzy sets* have been introduced (see [29]), where the membership degree of an element can be any subset of [0,1]. All previous tools are mainly used in the management of imprecise quantitative information. However, experts deal in many problems with qualitative information usually expressed through linguistic terms whose meaning is imprecise and vague in general, leading to the fuzzy linguistic approach.

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http://dx.doi.org/10.1016/j.fss.2013.09.001

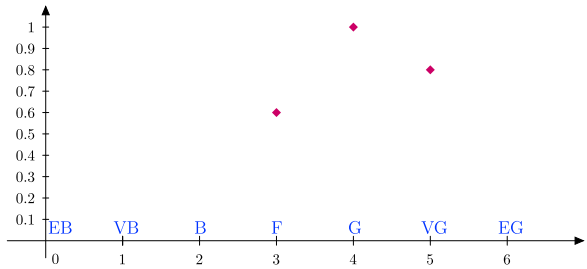

Fig. 1. Graphical representation of a discrete fuzzy number whose support is the interval [3, 5]. In addition, note that this fuzzy set can be interpreted as expression "between Fair and Very Good", after identifying the linguistic scale $\mathcal{L}$ with the chain L_6.

Qualitative information is often interpreted to take values in a totally ordered finite scale like

$$\{Extremely\ Bad,\ Very\ Bad,\ Bad,\ Fair,\ Good,\ Very\ Good,\ Extremely\ Good\}. \tag{1}$$

In these cases, the representative finite chain $L_n = \{0, 1, \ldots, n\}$ is usually considered to model these linguistic hedges and several researchers have developed an extensive study of aggregation functions on L_n, usually called *discrete aggregation functions* (see [16,19,21]). Another approximation is based on assigning a fuzzy set to each linguistic term trying to capture its meaning. However, the modeling of linguistic information is limited because the information provided by experts for each variable must be expressed by a simple linguistic term. In most cases, this is a problem for experts because their opinion does not agree with a concrete term. On the contrary, experts' values are usually expressions like "*better than Good*", "*between Fair and Very Good*" or even more complex expressions.

To avoid the limitation above, two approaches have recently appeared. [1] Both approaches are different in the form but very similar in the idea: they try to increase the flexibility of the elicitation of linguistic information.

- In [25] the concept of *Hesitant fuzzy linguistic term set* (HFLTS) is introduced in order to perform linguistic expressions like those mentioned in the previous paragraph. Basically, an HFLTS is simply any subset of the selected linguistic scale (usually subintervals). Thus, the expression "*between Fair and Very Good*" can be interpreted by the subinterval of the scale $\mathcal{L}$ given in (1):

  $$[Fair, Very\ Good] = \{Fair, Good, Very\ Good\}. \tag{2}$$

  Thus, all these expressions can be manipulated through HFLTS in a similar way to common Hesitant fuzzy sets (see [29]). In particular, aggregation operators like *Min-upper* and *Max-lower* operators are used in this context and applied in multicriteria linguistic decision making problem.
- The approach in [6,24] deals with the possibility of extending monotonic operations on L_n to operations on the set of discrete fuzzy numbers whose support is a set of consecutive natural numbers contained in L_n, usually denoted by $\mathcal{A}_1^{L_n}$. Here the idea lies on the fact that any discrete fuzzy number $A \in \mathcal{A}_1^{L_n}$ can be considered (identifying the scale $\mathcal{L}$ given in (1) with L_n with $n = 6$) as an assignation of a $[0, 1]$-value to each term in our linguistic scale. As an example, the above-mentioned expression "*between Fair and Very Good*" can be performed, for instance, by a discrete fuzzy number $A \in \mathcal{A}_1^{L_6}$, with support given by the subinterval stated in (2) (that corresponds to the subinterval $[3, 5]$ in L_6). The values of A in its support should be described by experts, allowing in this way a complete flexibility of the qualitative valuation. A possible discrete fuzzy number A representing the expression mentioned above is given in Fig. 1 (note that only the values of A in its support are pictured there).
  Thus, aggregation functions on $\mathcal{A}_1^{L_n}$ will allow us to manage qualitative information in a more flexible way. In [6] t-norms and t-conorms on $\mathcal{A}_1^{L_n}$ are described and studied, as well as it is done for uninorms, nullnorms and general aggregation functions in [24]. In both cases, an example of application in decision making or subjective evaluation is included.

[1] There are also other approaches that try to deal directly with linguistic expressions instead of single terms, see for instance [18,26].