Available online at www.sciencedirect.com

ScienceDirect

ELSEVIER

Fuzzy Sets and Systems 181 (2011) 88–101

FUZZY sets and systems

www.elsevier.com/locate/fss

Triangular norms which are meet-morphisms in interval-valued fuzzy set theory

Glad Deschrijver*

Fuzziness and Uncertainty Modeling Research Unit, Department of Applied Mathematics and Computer Science, Ghent University, Krijgslaan 281 (S9), B-9000 Gent, Belgium

Received 27 August 2010; received in revised form 27 May 2011; accepted 27 May 2011
Available online 7 June 2011

Abstract

In this paper we study t-norms on the lattice of closed subintervals of the unit interval. Unlike for t-norms on a product lattice for which there exists a straightforward characterization of t-norms which are join-morphisms, respectively meet-morphisms, the situation is more complicated for t-norms in interval-valued fuzzy set theory. In previous papers several characterizations were given of t-norms in interval-valued fuzzy set theory which are join-morphisms and which satisfy additional properties, but little attention has been paid to meet-morphisms. Therefore, in this paper, we focus on t-norms which are meet-morphisms. We consider a general class of t-norms and investigate under which conditions t-norms belonging to this class are meet-morphisms. We also characterize the t-norms which are both a join- and a meet-morphism and which satisfy an additional border condition.

Keywords: Interval-valued fuzzy set; t-Norm; Meet-morphism

1. Introduction

Interval-valued fuzzy set theory [11,15] is an extension of fuzzy theory in which to each element of the universe a closed subinterval of the unit interval is assigned which approximates the unknown membership degree. Another extension of fuzzy set theory is intuitionistic fuzzy set theory introduced by Atanassov [1]. In [8] it is shown that the underlying lattice of intuitionistic fuzzy set theory is isomorphic to the underlying lattice $\mathcal{L}^I$ of interval-valued fuzzy set theory.

In [6,7,5,18] several characterizations of t-norms on $\mathcal{L}^I$ in terms of t-norms on the unit interval are given. In [13,19,20] t-norms on related and more general lattices are investigated. However all the characterizations in these papers only deal with t-norms which are join-morphisms. Unlike for t-norms on a product lattice for which there exists a straightforward characterization of t-norms which are join-morphisms [3], respectively meet-morphisms, the situation is more complicated for t-norms in interval-valued fuzzy set theory. Therefore, in this paper, we focus on t-norms which are meet-morphisms. We consider a general class of t-norms (given in [7]) and investigate under which conditions t-norms belonging to this class are meet-morphisms.

* Tel.: +32 9 264 47 66; fax: +32 9 264 49 95.
E-mail address: Glad.Deschrijver@UGent.be
URL: http://www.fuzzy.UGent.be.

0165-0114/$ - see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.fss.2011.05.025

2. The lattice $\mathcal{L}^I$

Definition 2.1. We define $\mathcal{L}^I = (L^I, \leq_{L^I})$, where

$$L^I = \{[x_1, x_2] | (x_1, x_2) \in [0,1]^2 \text{ and } x_1 \leq x_2\},$$

$$[x_1, x_2] \leq_{L^I} [y_1, y_2] \iff (x_1 \leq y_1 \text{ and } x_2 \leq y_2), \text{ for all } [x_1, x_2], [y_1, y_2] \text{ in } L^I.$$

Similarly as Lemma 2.1 in [8] it can be shown that $\mathcal{L}^I$ is a complete lattice.

Definition 2.2 (*Gorzałczany [11] and Sambuc [15]*). An interval-valued fuzzy set on U is a mapping $A : U \to L^I$.

Definition 2.3 (*Atanassov [1]*). An intuitionistic fuzzy set on U is a set

$$A = \{(u, \mu_A(u), \nu_A(u)) | u \in U\},$$

where $\mu_A(u) \in [0,1]$ denotes the membership degree and $\nu_A(u) \in [0,1]$ the non-membership degree of u in A and where for all $u \in U$, $\mu_A(u) + \nu_A(u) \leq 1$.

An intuitionistic fuzzy set A on U can be represented by the $\mathcal{L}^I$-fuzzy set A given by

$$A \ : \ U \to L^I :$$
$$u \mapsto [\mu_A(u), 1 - \nu_A(u)].$$

In Fig. 1 the set L^I is shown. Note that to each element $x = [x_1, x_2]$ of L^I corresponds a point $(x_1, x_2) \in \mathbb{R}^2$.

In the sequel, if $x \in L^I$, then we denote its bounds by x_1 and x_2, i.e. $x = [x_1, x_2]$. The length $x_2 - x_1$ of the interval $x \in L^I$ is called the degree of uncertainty and is denoted by x_π. The smallest and the largest elements of $\mathcal{L}^I$ are given by $0_{\mathcal{L}^I} = [0, 0]$ and $1_{\mathcal{L}^I} = [1, 1]$. Note that, for x, y in L^I, $x <_{L^I} y$ is equivalent to $x \leq_{L^I} y$ and $x \neq y$, i.e. either $x_1 < y_1$ and $x_2 \leq y_2$, or $x_1 \leq y_1$ and $x_2 < y_2$. We define for further usage the set $D = \{[x_1, x_1] | x_1 \in [0, 1]\}$.

Note that for any non-empty subset A of L^I it holds that

$$\sup A = [\sup\{x_1 | [x_1, x_2] \in A\}, \sup\{x_2 | [x_1, x_2] \in A\}],$$

$$\inf A = [\inf\{x_1 | [x_1, x_2] \in A\}, \inf\{x_2 | [x_1, x_2] \in A\}].$$

Theorem 2.1 (*Characterization of supremum in $\mathcal{L}^I$, Deschrijver et al. [6]*). *Let A be an arbitrary non-empty subset of L^I and $a \in L^I$. Then $a = \sup A$ if and only if*

$$(\forall x \in A)(x \leq_{L^I} a)$$

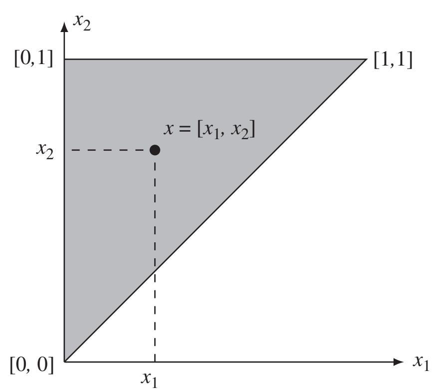

Fig. 1. The grey area is L^I.