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Stackelberg solutions for fuzzy random two-level linear programming through probability maximization with possibility

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Abstract

This paper considers Stackelberg solutions for decision making problems in hierarchical organizations under fuzzy random environments. Taking into account vagueness of judgments of decision makers, fuzzy goals are introduced into the formulated fuzzy random two-level linear programming problems. On the basis of the possibility and necessity measures that each objective function fulfills the corresponding fuzzy goal, together with the introduction of probability maximization criterion in stochastic programming, we propose new two-level fuzzy random decision making models which maximize the probabilities that the degrees of possibility and necessity are greater than or equal to certain values. Through the proposed models, it is shown that the original two-level linear programming problems with fuzzy random variables can be transformed into deterministic two-level linear fractional programming problems. For the transformed problems, extended concepts of Stackelberg solutions are defined and computational methods are also presented. A numerical example is provided to illustrate the proposed methods.

Keywords: Two-level linear programming; Fuzzy random variable; Stackelberg solutions; Possibility; Necessity; Probability maximization

1. Introduction

Decision making problems in decentralized organizations are often modeled as Stackelberg games [46], and they are formulated as two-level mathematical programming problems [45,44]. In the context of two-level programming, the decision maker at the upper level first specifies a strategy, and then the decision maker at the lower level specifies a strategy so as to optimize the objective with full knowledge of the action of the decision maker at the upper level. In conventional multi-level mathematical programming models employing the solution concept of Stackelberg equilibrium, it is assumed that there is no communication among decision makers, or they do not make any binding agreement even if there exists such communication. Computational methods for obtaining Stackelberg solutions to two-level linear programming problems are classified roughly into three categories: the vertex enumeration approach [5], the Kuhn–Tucker approach [3–5,16], and the penalty function approach [52]. The subsequent works on two-level programming problems under noncooperative behavior of the decision makers have been appearing [33–35,15,10,12] including some applications to aluminium production process [32], pollution control policy determination [2], tax credits determination for biofuel producers [11], pricing in competitive electricity markets [13], supply chain planning [39] and so forth.

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However, to utilize two-level programming for resolution of conflict in decision making problems in real-world decentralized organizations, it is important to realize that simultaneous considerations of fuzziness [41–43] and randomness [47,6] would be required. Fuzzy random variables, first introduced by Kwakernaak [25], have been developing [24,37,28], and an overview of the developments of fuzzy random variables was found in [14]. Studies on linear programming problems with fuzzy random variable coefficients, called fuzzy random linear programming problems, were initiated by Wang and Qiao [50], Qiao et al. [38] as seeking the probability distribution of the optimal solution and optimal value. Optimization models for fuzzy random linear programming problems were first considered by Luhandjula et al. [29,31] and further developed by Liu [26,27] and Rommelfanger [40]. A brief survey of major fuzzy stochastic programming models was found in the paper by Luhandjula [30]. As we look at recent developments in the fields of fuzzy random programming, we can see continuing advances [17–23,40,1,53].

Under these circumstances, in this paper, we consider Stackelberg solutions for decision making problems in hierarchical organizations under fuzzy random environments. Taking into account vagueness of judgments of decision makers, fuzzy goals are introduced into the formulated noncooperative two-level linear programming problems involving fuzzy random variables. After considering the possibility and necessity measures that each objective function fulfills the corresponding fuzzy goal, through the use of probability maximization [9] in stochastic programming, we propose new two-level fuzzy random decision making models which maximize the probabilities that the degrees of possibility and necessity are greater than or equal to certain values.

For the transformed problems, extended concepts of Stackelberg solutions are introduced and computational methods are also presented. It is shown that extended Stackelberg solutions can be obtained through the combined use of the variable transformation method by Charnes et al. [9] and the Kth best algorithm for two-level linear programming problems by Bialas et al. [5].

2. Fuzzy random two-level linear programming

A fuzzy random variable was first introduced by Kwakernaak [25], and its mathematical basis was constructed by Puri and Ralescu [37]. An overview of the developments of fuzzy random variables was found in the recent article of Gil et al. [14].

In general, fuzzy random variables can be defined in an n dimensional Euclidian space $\mathcal{R}^n$ [37]. From a practical viewpoint, as a special case of the definition by Puri and Ralescu, following the definition by Wang and Zhang [51], we present the definition of a fuzzy random variable in a single dimensional Euclidian space $\mathcal{R}$.

Definition 1 (*Fuzzy random variable*). Let $(\Omega, \mathcal{A}, P)$ be a probability space, where Ω is a sample space, $\mathcal{A}$ is a σ-field and P is a probability measure. Let F_N be the set of all fuzzy numbers and $\mathcal{B}$ a Borel σ-field of $\mathcal{R}$. Then, a map $\tilde{\bar{C}} : \Omega \to F_N$ is called a fuzzy random variable if it holds that

$$\{(\omega, \tau) \in \Omega \times \mathcal{R} | \tau \in \tilde{\bar{C}}_\alpha(\omega)\} \in \mathcal{A} \times \mathcal{B}, \quad \forall \alpha \in [0, 1], \tag{1}$$

where $\tilde{\bar{C}}_\alpha(\omega) = [\tilde{\bar{C}}_\alpha^-(\omega), \tilde{\bar{C}}_\alpha^+(\omega)] = \{\tau \in \mathcal{R} | \mu_{\tilde{\bar{C}}(\omega)}(\tau) \geq \alpha\}$ is an α-level set of the fuzzy number $\tilde{\bar{C}}(\omega)$ for $\omega \in \Omega$.

Intuitively, fuzzy random variables are considered to be random variables whose realized values are not real values but fuzzy numbers or fuzzy sets.

2.1. Problem formulation

In this paper, we deal with on two-level linear programming problems involving fuzzy random variable coefficients in objective functions formulated as

$$\left.\begin{array}{ll} \underset{\text{for DM1}}{\text{minimize}} & z_1(\boldsymbol{x}_1, \boldsymbol{x}_2) = \tilde{\bar{\boldsymbol{C}}}_{11}\boldsymbol{x}_1 + \tilde{\bar{\boldsymbol{C}}}_{12}\boldsymbol{x}_2 \\ \underset{\text{for DM2}}{\text{minimize}} & z_2(\boldsymbol{x}_1, \boldsymbol{x}_2) = \tilde{\bar{\boldsymbol{C}}}_{21}\boldsymbol{x}_1 + \tilde{\bar{\boldsymbol{C}}}_{22}\boldsymbol{x}_2 \\ \text{subject to} & A_1\boldsymbol{x}_1 + A_2\boldsymbol{x}_2 \leq \boldsymbol{b} \\ & \boldsymbol{x}_1 \geq \boldsymbol{0}, \quad \boldsymbol{x}_2 \geq \boldsymbol{0} \end{array}\right\}. \tag{2}$$