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On the calculation of a membership function for the solution of a fuzzy linear optimization problem

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Abstract

In the present paper the fuzzy linear optimization problem (with fuzzy coefficients in the objective function) is considered. Recent concepts of fuzzy solution to the fuzzy optimization problem based on the level-cut and the set of Pareto optimal solutions of a multiobjective optimization problem are applied. Chanas and Kuchta suggested one approach to determine the membership function values of fuzzy optimal solutions of the fuzzy optimization problem, which is based on calculating the sum of lengths of certain intervals. The purpose of this paper is to determine a method for realizing this idea. We derive explicit formulas for the bounds of these intervals in the case of triangular fuzzy numbers and show that only one interval needs to be considered.

Keywords: Fuzzy optimization; Multicriteria optimization; Pareto optimal solution; Membership function

1. Introduction

In many situations optimization problems with unknown or only approximately known data need to be solved e.g. the fuzzy multicommodity flow problem (compute optimal flows in a traffic network with fuzzy costs for passing streets, see e.g. [1]) or problems of optimal planning [2]. It seems reasonable to approach these problems within the framework of fuzzy set theory because continuous fuzzy numbers are particularly suited for describing such ambiguities.

Let us formulate a fuzzy optimization problem where the objective function has fuzzy values and the constraint function is a crisp one, i.e.:

$$\begin{aligned} &\widetilde{f}(x) \to \min, \\ &g(x) \leq 0. \end{aligned} \tag{1}$$

Here $g = (g_1, \ldots, g_k) : \mathbb{R}^n \to \mathbb{R}^k$ is a crisp function and $\widetilde{f}$ maps $\mathbb{R}^n$ to the space of fuzzy numbers.

The formulation of fuzzy optimization problems with no objective but fuzzy constraints can be found in [3,4].

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The linear case of (1) is represented by the following linear programming problem with fuzzy coefficients in the objective function:

$$\tilde{c}^\top x \to \min,$$
$$Ax \le b,$$
$$x \ge 0. \tag{2}$$

Many authors try to find a *single* best solution of the fuzzy optimization problem (see e.g. [5,6]). Those approaches are based on the extension principle suggested by Bellman and Zadeh [7]. We suggest to reflect the uncertainty in fuzzy optimization problems through (the existence of) a set of optimal solutions. Under the assumption that this set consists of more than one element, the decision-maker can improve a choice relying on some criteria that are not a priory defined in the optimization problem.

When the fuzzy optimization problem is solved it is natural to consider its solutions as fuzzy. Hence, a criterion for comparing the elements of the fuzzy set of optimal solutions of the fuzzy optimization problem is required. One possible choice is to compare values of the corresponding membership functions. One approach to calculate such a membership function is suggested by Chanas and Kuchta [8,9]. Knowledge of the membership function values of the elements of the set of fuzzy optimal solutions enables the decision-maker to make an *educated* choice between these solutions. Moreover, using our approach, a decision-maker can see a correlation among solutions and quantitatively measure the advantage of his choice over other solutions. The main aim of the present paper is to find the best realization of this idea based on modern solution algorithms [6,8–13].

To describe the membership function of the fuzzy linear objective function, α-cuts are used. It is assumed that its left- and right-hand side values are given by the functions $c_L(\alpha)$ and $c_R(\alpha)$ for $\alpha \in [0, 1]$. Following [14] we assume that $c_L(\alpha)$ is a bounded increasing function and $c_R(\alpha)$ is a bounded decreasing function of α. Moreover, it is obvious that $c_L(\alpha) \le c_R(\alpha)$ for all $\alpha \in [0, 1]$.

Then, using a suitable ordering of the intervals $\tilde{c}[\alpha] := [c_L(\alpha), c_R(\alpha)]$ for fixed level-cuts, the task of the fuzzy function minimization over a feasible set is transformed into a bicriterial optimization problem, which is solved by means of the scalarization technique.

Clearly, the approach of partial ordering the intervals may lead to situations of indecisiveness [15]. This reflects the noncomparability of the elements of the set of Pareto optimal solutions of the biobjective optimization problem. The computation of all such solutions is the basis of our approach to compute the membership function values of the fuzzy solutions of the initial problem. For this, we have to compute the membership function values for all feasible points with the presented algorithm.

As soon as a solution of the scalarized problem (with fixed α-cut) depends on a parameter λ, a variation of $\lambda \in [0, 1]$ gives the optimal solution points. The set of those points then represents a subset of a Pareto set. The solution of the fuzzy linear optimization problem (2) is defined through the Pareto-optimal solution of a bicriterial optimization problem. This will be described in Section 2.

In Section 3 optimality conditions for the fuzzy linear optimization problem (2) are derived. A procedure to compute the membership function of each fuzzy solution is given in Section 4. For brevity, the discussions is limited to one element of the set of fuzzy optimal solutions, but can easily be extended.

The paper is concluded with a short discussion devoted to the wide class of triangular fuzzy numbers (that can be extended to the class of LR-numbers) in Section 5 and a numerical example in Section 6.

2. Fuzzy linear optimization problem

For simplicity instead of (2) we investigate the fuzzy linear optimization problem:

$$\tilde{c}^\top x \to \min,$$
$$Ax = b,$$
$$x \ge 0 \tag{3}$$