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Theoretical and semantic distinctions of fuzzy, possibilistic, and mixed fuzzy/possibilistic optimization

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Abstract

Theoretical, semantic, and algorithmic distinctions among fuzzy, possibilistic and mixed fuzzy/possibilistic optimization are presented and illustrated. The theory underlying fuzzy, possibilistic, and mixed fuzzy/possibilistic optimization is developed and demonstrated and points to the appropriate use of distinct solution methods associated with each type of optimization dependant on the semantics of the problem. Semantics is key to both the input where one is obtaining the data and constructing the optimization model in the presence of uncertainty and the output where the meaning of the results is necessary for understanding solutions. The case in which the optimization model arises from the data that is a combination of fuzzy and possibilistic distributions is also derived. Lastly, examples illustrate the theory. This paper is a modification and an amplification of a presentation made at IFSA'05 [W.A. Lodwich, K.D. Jamison, Theory and semantics for fuzzy and possibilistic optimization, in: Fuzzy Logic, Soft Computing and Computational Intelligence, Eleventh Internat. Fuzzy Systems Association World Congress, July 28–31, 2005, Beijing, China, Vol. III, pp. 1805–1810 [26]].

Keywords: Fuzzy optimization; Possibilistic optimization; Linear and nonlinear programming

1. Introduction

Uncertainty optimization problems are among the hardest to solve because the meanings of inequalities and optima must be defined in the context of the problem in question. Moreover, the complexity of uncertain optimization is generally great. While the present paper focuses on theory and semantics of fuzzy and possibilistic optimization, it should be noted that many of the ideas contained herein have been applied to real large-scale problems (see [24]). This research makes the case for separating fuzzy and possibilistic optimization both in terms of semantics as well as in computational methods and develops approaches of how to deal with the case in which both fuzzy and possibilistic uncertainties appear in the same model. We assume that the reader is familiar with fuzzy set theory, possibilistic theory, and mathematical programming.

This paper is organized as follows. This first introductory section contains the discussion of the general problem of optimization under uncertainty. The second section contains the discussion of fuzzy, possibilistic, and mixed fuzzy/possibility semantics in the context of optimization models. The third section discusses solution methods for

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fuzzy, possibilistic, and mixed fuzzy/possibilistic optimization along with solution semantics. Conclusions are found in the last section.

1.1. Optimization in the presence of uncertainty

Optimization under uncertainty, as used here, means optimization when at least one element of the input data is a real-valued interval, a real-valued random variable, a real-valued fuzzy number, or a real-valued number described by a possibility/necessity distribution. This paper focuses on fuzzy and possibilistic uncertainty and its associated optimization models. The use of necessity distributions are done similarly to possibility except that the necessity sematic is a pessimistic one while the possibility semantic is optimistic. The methods developed map the uncertainty onto the set of real numbers rather than dealing with partial ordering of distributions. Obtaining an order on distributions is the domain of stochastic dominance theory (see [13,23,30,34]). While relevant, it is beyond the scope of this study.

1.2. Sources of uncertainty

We consider the following general programming problem:

$$\begin{aligned} z = \min \quad & f(x,a) \\ \text{s.t.} \quad & g_i(x,b) \leqslant 0, \quad i = 1,\ldots,M_1, \\ & h_j(x,c) = 0, \quad j = 1,\ldots,M_2, \\ & x \in S. \end{aligned} \tag{1}$$

The constraint set is denoted $\Omega = \{x \mid g_i(x,b) \leqslant 0 \; i = 1,\ldots,M_1, h_j(x,c) = 0 \; j = 1,\ldots,M_2, x \in S\}$. It is assumed that $\Omega \neq \{\emptyset\}$. The values of a, b, and c are inputs (data, coefficients, and parameters) of the programming problem. These values are subject to uncertainty for a variety of reasons. Depending on the nature of the uncertainty, they may be probability distributions, intervals, fuzzy sets, or possibilistic distributions. Moreover, the operator min and relationships $=$, $\leqslant$ and $\in$ can take on a flexible or fuzzy meaning becoming a soft relationship or constraint. For example, the equality and inequality relationships may be aspirations, that is, they may take on the meaning, "Come as close as possible to satisfying the relationships with some degree of violation being permissible." On the other hand, the values of a, b, or c may be described by a probability, interval, fuzzy, or possibilistic distribution. In either case (uncertainty in the relationship, uncertainty in the parameters) the meaning of the relationships must be specified. It is noted that when the objective function and/or constraints are uncertain, the optimization problem may not be (undoubted is not) convex so that the usual solution methods are local. In very simple cases where the constraint is of the form $Ax - b \leqslant 0$, and the coefficients of the matrix are intervals, the solution set can be a star-shaped region (see [12]). Recall that an interval is a fuzzy number. Moreover, uncertainty in the matrix A means that the underlying model as specified by linear relationships is not known exactly or that the model is precise but knowledge of what the value of the data are, is incomplete.

2. Fuzzy and possibilistic optimization models: semantics

There is often confusion about fuzzy and possibilistic optimization. Fuzzy and possibilistic entities have different meanings, semantics. Fuzzy and possibility uncertainty model different entities and the associated solution methods are different as we shall see. Fuzzy entities, as is well known, are sets with non-sharp boundaries in which there is a transition between elements that belong and elements that do not belong to the set. Possibilistic entities are real-valued entities that exist, but the evidence associated with whether or not a particular element belongs to the set is incomplete or hard to obtain. We use a tilde, $\sim$, to denote a fuzzy set and a "hat", $\wedge$, to denote a possibility distribution.

2.1. Fuzzy and possibilistic optimization semantics

Next what is meant by decision-making in the presence of fuzzy and possibilistic entities is defined. These definitions are central to the semantics and methods. In their book Dubois and Prade [5, Chapter 5] give clear definitions and