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Qualitative optimization of Fuzzy Causal Rule Bases using Fuzzy Boolean Nets[☆]

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Abstract

Fuzzy Causal Rule Bases (FCRb) are widely used and are the most important rule bases in Rule Based Fuzzy Cognitive Maps (RB-FCM). However, FCRb are subject to several restrictions that create difficulties in their creation and completion. This paper proposes a method to optimally complete FCRb using Fuzzy Boolean Net properties as qualitative universal approximators. Although the proposed approach focuses on FCRb, it can be generalized to any fuzzy rule base.

Keywords: Fuzzy Boolean Nets; Fuzzy Causal Relations; Rule base optimization

1. Introduction

Rule Based Fuzzy Cognitive Maps (RB-FCM) are a qualitative approach to modelling and simulating the dynamics of qualitative systems (like, for instance, social, economical or political systems) [3,5,8]. RB-FCM were developed as a tool that can be used by non-engineers and/or non-mathematicians and eliminates the need for complex mathematical knowledge when modelling qualitative dynamic systems. Fuzzy Causal Relations (FCR) were previously introduced in [1,2,6], and are the most common method to describe the relations between the entities (known as concepts) of RB-FCM [1,2,6]. FCR are represented and defined through linguistic Fuzzy Causal Rule Bases (FCRb). RB-FCM inference imposes that FCRb must be complete and involve only one antecedent (multiple antecedent inference is dealt with internal RB-FCM mechanisms, like the Fuzzy Causal Accumulation operation [1,2,5]). It also imposes certain strict restrictions to the linguistic terms involved in the inference [1,2,6] (Section 2). Another important characteristic of FCRb is the unusually large number of linguistic terms needed to properly represent the involved relations in typical applications (variables with 11 or 13 linguistic terms are common in FCRb) [5,8].

FCR data are usually obtained through “far from ideal” methods, and all of the above characteristics and restrictions mean that extra-special care must be taken with FCRb construction when modelling a RB-FCM, especially when one considers that RB-FCM usually contain a large number of feedback cycles, and as a consequence, minor differences

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in modelling result in drastic simulation differences and conclusions [4,8,18]. Therefore, FCR data must often be optimized before it can be used on the RB-FCM. As we will see next, this optimization process differs from classical optimization problems due to several factors.

On a RB-FCM, experts usually express knowledge using just a few key rules. These key rules should contain enough knowledge for a human expert to extrapolate all the rules necessary to complete the FCRb. An optimal FCRb consists on the set of rules extrapolated by an expert from the available key rules. In FCRb such set must be complete (see Section 2) and usually has a cardinality of 11 or 13 [5,8]. Under this definition, the optimal FCRb have several characteristics that prevent the use of a classic quantitative optimization approach. The problem of obtaining the optimal set lies in the fact that what is relatively easy for a human is not necessarily easily automated when one deals with qualitative linguistic knowledge representation. For example, a simple linear relation between two linguistic fuzzy variables cannot necessarily be represented by a quantitative linear expression. A qualitative linear relation is usually expressed by an expert using just two fuzzy "if...then" rules (just as one would only need two points to define a straight line on a quantitative problem). However, if the membership functions used to define the linguistic terms are not identical and/or are not regularly distributed over the Universe of Discourse (UoD), we are indeed in presence of a quantitative non-linear relation that possibly cannot be optimally approached using a linear function. Note that in the simplest case, where the number of antecedent and consequent linguistic terms is identical, finding the optimal linguistic FCRb is trivial for an expert (see Section 5, Example 2), while a numerical approach can be far from trivial due to the non-linear constraints imposed by membership function shape, size and centre of gravity. Therefore optimal rule base completion should probably focus on alternative qualitative methods.

An additional difficulty concerning FCRb optimal completion lies in the fact there is no way to always objectively define an optimal solution, although that is possible in some particular cases (like when the expert is defining a qualitative linear relation and the number of antecedent and consequent linguistic terms is identical). In fact, experts often provide a small number of rules even when expressing relations that are far from linear. Humans can usually still complete the FCRb using common sense (a quantitative 2D analogy would be imagining a line smoothly linking a few key points), but only the original expert would know for sure if the completion is optimal. In these cases, one simply cannot define an objective function to minimize, since only the expert that gave the key rules could properly validate a result. The best solution available is to make a "smooth" qualitative interpolation using as constraints the available key rules and the linguistic term set (the 2D analogy optimal solution would be the interpolation that gives the "smoothest line" and passes by a set of predefined points). Thus, for this study, an optimal result is obtained by finding the smoothest qualitative interpolation that respects all constraints.

The problem of optimal FCRb completion can become even more complex when one considers the cases where data come from more than one expert or from quantitative measurements. The optimization process should therefore be able to cope with the following situations:

- *Expert knowledge*
  - *Single expert case*: The expert usually expresses knowledge using just a few key rules that must be generalized to all UoD—one must find an optimal FCRb using a completion process.
  - *Multiple expert case*: Different experts might have different opinions on the same problems, and, as a result, supplied key rules can be different—one must find an optimal FCRb by combining different and possibly inconsistent rules, and by rule base completion.
- *Uncertain and sparse quantitative data*: In some cases the need arises to obtain an FCRb from observations, measurements, etc.—finding an optimal rule base involves qualitative rule extraction from data followed by rule base completion.

Several methods have been proposed to address the above (or part of the above) problems. However, although those methods are valid in most fuzzy rule base completion problems, they fall short when dealing with FCR optimization for several reasons we present in the next section.

To solve the problem of optimal FCRb completion, we propose the use of Fuzzy Boolean Nets (FBN). FBN have been previously introduced as a hybrid fuzzy-neural technique where fuzziness is an emergent property that gives FBN the capability of extracting qualitative fuzzy rules from quantitative data [20,21,23]. FBN are qualitative universal approximators [19] with an excellent generalization capability that allows them to perform as qualitative interpolators. It is our contention that FBN produce an optimal completion because they generate the "best" and most robust smooth qualitative rule base.