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Computing efficient solutions to fuzzy multiple objective linear programming problems

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Abstract

In this paper, we improve the fuzzy compromise approach of Guu and Wu by automatically computing proper membership thresholds instead of choosing them. Indeed, in practice, choosing membership thresholds arbitrarily may result in an infeasible optimization problem. Although we can adjust minimum satisfaction degree to get fuzzy efficient solution, it sometimes makes the process of interaction more complicated. In order to overcome this drawback, a theoretically and practically more efficient two-phase max–min fuzzy compromise approach is proposed in this paper. Moreover, the efficiency of the two-phase approach is verified by an example.

Keywords: Fuzzy multiple objective linear programming; Max–min operator approach; Fuzzy compromise approach; Two-phase approach

1. Introduction

Multiple Objective Linear Programming (MOLP) problems can be formulated as:

$$\begin{array}{ll} \max & Z = [c_1x, \ldots, c_Nx]^{\mathrm{T}} = [Z_1(x), \ldots, Z_N(x)]^{\mathrm{T}} \\ \text{s.t.} & x \in X, \quad X = \{x \in R^n\colon\ Ax \leqslant b,\ x \geqslant 0\} \end{array} \tag{1}$$

where $A = (a_{ij})_{m\times n}$, $c_i \in R^n$ $(0 \leqslant i \leqslant N)$, $b \in R^m$. In problem (1), all objective functions can hardly reach their optima at the same time subject to the given constraints. Therefore, in practice the decision-maker chooses some efficient solutions as final decision according to the satisfaction degree (or preference) of each objective value. The fuzzy approach for solving MOLP proposed by Zimmermann [10] has given an effective way of measuring the satisfaction degree of MOLP.

Definition 1.1. The vector, whose components are composed by maximum value of each objective function under the given constraints, i.e.

$$Z^* = [Z_1^*, \ldots, Z_N^*] = [\max(Z_1(x)), \ldots, \max(Z_N(x))] \tag{2}$$

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is called the ideal solution [1]. Similarly, the negative ideal solution is defined by

$$Z^- = [Z_1^-, \dots, Z_N^-] = [\min(Z_1(x)), \dots, \min(Z_N(X))]. \tag{3}$$

Let the initial solution of objective vector given by decision-maker be

$$O = [O_1, \dots, O_N]. \tag{4}$$

To make a proper decision on the choice of initial solution, a decision-maker can use the negative ideal solution as a reference point, namely, chooses the initial solution not less than negative ideal solution. Furthermore, membership function [5] of each objective function value's satisfaction degrees can be defined as the following:

$$u_k(x) = \begin{cases} 1, & Z_k(x) > Z_k^*, \\ 1 - \dfrac{Z_k^* - Z_k(x)}{Z_k^* - O_k} & O_k < Z_k(x) \leqslant Z_k^*, \qquad k = 1, \dots, N. \\ 0, & Z_k(x) \leqslant O_k, \end{cases} \tag{5}$$

If the initial solution is chosen to be negative ideal solution, then membership function of each objective value's satisfaction degrees is given by

$$u_k^-(x) = \begin{cases} 1, & Z_k(x) > Z_k^*, \\ 1 - \dfrac{Z_k^* - Z_k(x)}{Z_k^* - Z_k^-}, & Z_k^- < Z_k(x) \leqslant Z_k^*, \qquad k = 1, \dots, N. \\ 0, & Z_k(x) \leqslant Z_k^-, \end{cases} \tag{6}$$

To solve problem (1), following the concept of membership function, Zimmermann proposed max–min operator approach [9]:

$$\begin{array}{ll} \max & \lambda \\ \text{s.t.} & \lambda \leqslant u_k(x), \quad k = 1, \dots, N, \\ & \lambda \in [0, 1], \quad x \in X. \end{array} \tag{7}$$

It has been proved that max–min operator approach possesses some good properties. However, the efficiency of the solution yielded by max–min operator is not guaranteed. In order to overcome inefficiency, in this paper, the compromise approach and two-phase approach are proposed (see [4,7,2]).

2. Fuzzy compromise method for MOLP

Based on the above analysis, problem (1) can be transformed into the following linear programming problem (fuzzy compromise approach [8]):

$$\begin{array}{ll} \max & \lambda = \sum\limits_{k=1}^{N} \omega_k \lambda_k \\ \text{s.t.} & \lambda_k^l \leqslant \lambda_k \leqslant u_k(x), \quad k = 1, \dots, N, \\ & x \in X, \quad \sum\limits_{k=1}^{N} \omega_k = 1, \quad \omega_k > 0, \end{array} \tag{8}$$

where λ_k^l $(0 \leqslant \lambda_k^l \leqslant 1)$ is the minimum satisfaction degree of the kth objective function chosen by decision-maker. To increase the minimum satisfaction degree of one objective function means that the value of this objective function is closer to the optimal value; on the other hand, it may make other objective function values far from their optimal values. As a result, when the minimum satisfaction degree chosen by decision-maker is too great, (8) may have no solution. So, to ensure the existence of a feasible solution, λ_k^l needs to be adjusted properly.

Definition 2.1. x^* is a fuzzy-efficient solution to (8) if there does not exist any $x \in X$ such that $u_k(x^*) \leqslant u_k(x)$ for all k $(k = 1, \dots, N)$ and $u_s(x^*) < u_k(x)$ for at least one s.