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# Learning of interval and general type-2 fuzzy logic systems using simulated annealing: Theory and practice

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ABSTRACT

This paper reports the use of simulated annealing to design more efficient fuzzy logic systems to model problems with associated uncertainties. Simulated annealing is used within this work as a method for learning the best configurations of interval and general type-2 fuzzy logic systems to maximize their modeling ability. The combination of simulated annealing with these models is presented in the modeling of four benchmark problems including real-world problems. The type-2 fuzzy logic system models are compared in their ability to model uncertainties associated with these problems. Issues related to this combination between simulated annealing and fuzzy logic systems, including type-2 fuzzy logic systems, are discussed. The results demonstrate that learning the third dimension in type-2 fuzzy sets with a deterministic defuzzifier can add more capability to modeling than interval type-2 fuzzy logic systems. This finding can be seen as an important advance in type-2 fuzzy logic systems research and should increase the level of interest in the modeling applications of general type-2 fuzzy logic systems, despite their greater computational load.



## 1. Introduction

Fuzzy logic systems have been applied successfully to a broad range of problems in different application domains. One such type of application is concerned with using fuzzy logic for system modeling and approximation where a fuzzy inference system is used to model human knowledge or to approximate non-linear and dynamic systems. However, the existence of uncertainties and lack of information in many real-world problems makes it difficult to model such problems using expert knowledge only. Examples of such problems include identifying systems with no known rule-base and systems with only historic data observation. It becomes clear that when designing a simple fuzzy logic system with few inputs, the experts may be able to provide efficient rules but, as the complexity of the system grows, suitable rule-base and membership functions become difficult to acquire. Therefore, some automated tuning and learning methods are often used to cope with such situations. The objective of these methods is to get parameterized functions that best model these problems according to chosen criteria. The use of automated methods to design fuzzy logic systems has helped to model many real-world problems

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that are difficult to understand by experts and it is now a well-established methodology for modeling and approximation applications. The *motivation* for this research is two-fold:

- Type-2 fuzzy logic systems have numerous parameters that need to be determined in the design of any system. The determination of these parameters is an open research question and motivates our approach to learning type-2 fuzzy systems.
- The growth in interest in type-2 fuzzy logic has not fully manifested itself in real-world applications using general type-2 fuzzy sets. The emphasis has been on interval type-2 fuzzy sets, thus not taking advantage of the more general representation. By allowing for the learning and optimization of type-2 fuzzy systems we expect the use of general type-2 fuzzy sets to grow.

So the motivation is clear, and we now elaborate on these points.

*Learning and optimization.* This research is concerned with the learning of type-2 fuzzy logic systems, both general and interval. Type-2 fuzzy logic systems are now well established as both a research topic and an application tool. The motivation for the use of type-2 fuzzy sets is that type-1 fuzzy logic has problems when faced with environments that contain uncertainties that are typical in a large number of real-world applications. These uncertainties in the environment translate into uncertainties about membership functions [38]. Type-1 fuzzy logic cannot fully handle these uncertainties because it is precise in nature and for many applications it is unable to model knowledge adequately, while type-2 fuzzy logic offers a higher level of imprecision modeling [26]. The extra dimension and parameters in type-2 fuzzy sets are supposed to provide more design freedom and flexibility than type-1 fuzzy sets. The use of automated learning methods becomes important as complexity grows when designing type-2 fuzzy logic systems.

Many approaches have been proposed to learn and tune type-1 and type-2 fuzzy logic systems including search algorithms such as genetic algorithms and particle swarm algorithms, as well as local search algorithms and classical learning methods. Compared to genetic algorithms, few researchers have studied use of simulated annealing to learn type-1 fuzzy logic systems such as [12,16,49]. So far as we are aware, the only research reported on the use of simulated annealing to design type-2 fuzzy logic systems is the authors' previous work in [3–6].

*Helping develop real-world applications.* Another motivation for this research comes from the lack of applications using general type-2 fuzzy logic systems. Type-2 fuzzy logic is a growing research topic with much evidence of successful applications. However, almost all developments of type-2 fuzzy logic systems have been based on interval type-2 fuzzy logic [27,45]. The heavy computational load associated with the generalized form of type-2 sets is the main driver for the lack of applications of general type-2 fuzzy sets compared with the interval model. This prior work has reinforced the common concept that interval type-2 fuzzy logic systems can add more modeling capabilities than type-1 fuzzy logic systems but with extra computational cost. Learning and optimization of general type-2 fuzzy logic systems are open areas for more research, as well as the ongoing research on how to reduce the complexity of general type-2 fuzzy logic systems, especially in the type-reduction phase of the system. The large number of methods used to design type-1 and interval type-2 fuzzy logic systems can be seen as potential candidates for general type-2 fuzzy logic systems and some of them might uncover further possibilities for modeling uncertainty. However, recent advances in general type-2 fuzzy logic systems research, including new representations, optimized operations and faster type-reduction methods, indicate an expected growth in applications. Despite the larger number of computations associated with general type-2 fuzzy sets, there may well be benefits compared to interval type-2 fuzzy sets. This ability can be unveiled using automated designing methods rather than being chosen by the designer manually. Automated methods can fine-tune initial fuzzy logic system designs due to the lack of a rational basis for choosing secondary membership functions for general type-2 fuzzy sets [36, p. 302]. This issue enforces the need for using automated methods in such problem. The other factor affecting the usage of general type-2 fuzzy logic systems is the lack of practical parameterization methods to handle the third dimension in general type-2 fuzzy sets. In general, a general type-2 fuzzy logic system has the potential to model more uncertainties despite the large amount of computations associated with it especially when applied to nonreal-time applications. In consequence, the question of how much general type-2 fuzzy logic systems can add to modeling performance over interval type-2 fuzzy logic systems is another issue that warrants investigation.

The research reported here introduces a new method for learning general type-2 fuzzy systems with a unique combination of learning the footprint of uncertainty (FOU) followed by learning the secondary membership functions (SMF). In addition, we show that when using the vertical slice type reducer we have improvement over other approaches implemented here. Furthermore, interval type-2 fuzzy logic systems were applied to answer the question of to what extent general type-2 fuzzy sets can add more abilities and flexibilities to modeling than interval type-2 fuzzy sets. A detailed analysis is carried out of the learning of general type-2 fuzzy systems on a set of real-world data with and without added noise and, as such, provides significant insight into how the future of learning general type-2 fuzzy systems can be carried out. These methods are applied to four benchmark problems: noise-free Mackey–Glass time series forecasting [34], noisy Mackey–Glass time series forecasting [34], and two real-world problems, namely the estimation of the low-voltage electrical line length in rural towns and the estimation of the medium-voltage electrical line maintenance cost [11].

The rest of this paper starts with a review of the methods and concepts used in this work in Section 2 and issues related to the design of general type-2 fuzzy logic systems in Sections 3 and 3.2. The methodology and the results are detailed in Sections 4 and 5 and some conclusions are drawn in Section 6.