Information Sciences 360 (2016) 96–111

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

L-concept analysis with positive and negative attributes ☆

Eduard Bartl*, Jan Konecny

Data Analysis and Modeling Lab, Department of Computer Science, Palacky University Olomouc, 17. listopadu 12, Olomouc CZ-77146, Czech Republic

ARTICLE INFO

Article history:
Received 26 March 2015
Revised 19 October 2015
Accepted 4 April 2016
Available online 26 April 2016

Keywords:
Antitone concept-forming operator
Concept lattice
Formal concept analysis
Galois connection
If-then rules
Isotone concept-forming operator

ABSTRACT

We describe an extension of formal fuzzy concept analysis allowing a user to choose which attributes are viewed as positive and which are viewed as negative. The two sets are then handled using a combination of previously studied antitone concept-forming operators and isotone concept-forming operators, respectively. The two main outputs of formal concept analysis, namely concept lattices and attribute implications, in the setting of positive and negative attributes are presented. An analogy of the main theorem of concept lattices and a relationship between the new concept lattice and the previously studied concept lattices is showed. We introduce basic syntactic and semantic notions for attribute implications called fuzzy containment implications. We consider two settings, one where the sets of positive and negative attributes are crisp sets, and a generalization, where the two sets are fuzzy sets.

1. Introduction

Formal concept analysis [9,12] is a method of relational data analysis identifying interesting clusters (formal concepts) in a collection of objects and their attributes, and organizing them into a structure called concept lattice. The formal concept is obtained as a fixed point of so-called concept-forming operators and is characterized by a pair of sets – extent and intent. The extent contains all objects covered by the concept and the intent contains all attributes covered by the concept. Numerous generalizations of formal concept analysis, which allow to work with graded data, were provided; see [16] and references therein. In the present paper we stick with approach of Belohlavek [2] and Pollandt [17].

In a graded (fuzzy) setting, two main kinds of concept forming-operators – antitone and isotone one – were studied [3,13,17,18], compared [5,6] and even covered under a unifying framework [4,15]. The antitone concept-forming operators handle attributes in a positive way and concepts are based on sharing attributes (at least in some degree), while the isotone concept-forming operators handle attributes in a negative way and concepts are based on missing same attributes (or having them at most in some degree).

In order to clarify our motivation, we assume the data (**L**-context) in Fig. 1 with objects representing employees, and attributes representing skills.

We consider the following four situations:

☆ Supported by Grant no. 15-17899S, "Decompositions of Matrices with Boolean and Ordinal Data: Theory and Algorithms", of the Czech Science Foundation.

* Corresponding author. Tel.: +420 585634720.
E-mail address: eduard.bartl@upol.cz (E. Bartl).

http://dx.doi.org/10.1016/j.ins.2016.04.012

	α	β	γ
A	0.5	0	1
B	1	0.5	1
C	0	0.5	0.5
D	0.5	0.5	1

Fig. 1. Example of **L**-context with objects A, B, C, D and attributes α, β, γ; **L** is a chain $0 < 0.5 < 1$ with Łukasiewicz operations.

(a) One can handle the attributes in positive way and form concepts based on having the same skills at least in some degree. Such concepts are formed by antitone concept-forming operators denoted by $\langle\uparrow, \downarrow\rangle$. Extents of the concepts can be interpreted as maximal collections of employees able to fulfill a task which requires particular skill set. For example, the collection of employees able to fulfill a task which requires the skill α in full degree and the skill β at least in half degree can be found as $\{\alpha, \deg 0.5\beta\}^{\downarrow}$.
(b) Or, one can handle the attributes in negative way and form concepts based on having the same skills at most in some degree. Such concepts are created by isotone concept-forming operators which we denote by $\langle\cap, \cup\rangle$. Extents of the concepts can be interpreted as maximal collections of employees who lack the same skills and need some training to gain them. For example, the maximal collection of employees who lack the skill α and have the skill β at most in degree 0.5 can be found as $\{\deg 0.5\beta, \gamma\}^{\cup}$.
(c) Now, consider a training course for the skill β for which is essential to have the skill α at least in degree 0.5. Concept covering just employees appropriate for the training course (i.e. employees who meet the requirement but have not mastered β yet) is not formed neither by antitone nor isotone concept-forming operators. While the attribute α is positive, β is negative and they must be handled in a different way.
(d) Finally, consider another training course to master the skill β for which is essential to already have the skill β in degree 0.5. In such a case, we are interested in a concept covering just employees having the skill β exactly in degree 0.5. In such a case, β is considered positively and negatively at the same time. Again, it cannot be formed neither by antitone nor isotone concept-forming operators.

This example motivates us to extend formal concept analysis in such a way that a user is allowed to specify a set ${}^{+}Y$ of positive attributes and a set ${}^{-}Y$ of negative attributes. Attributes in ${}^{+}Y$ and ${}^{-}Y$ are then handled using antitone concept-forming operators and isotone concept-forming operators, respectively. The present approach enables us to incorporate the concepts from (a) and concepts from (b) into one concept lattice, and to form concepts from (c) and (d).

We study the two main outputs of formal concept analysis, i.e. concept lattices and attribute implications, in the setting of positive and negative attributes.

We have considered a similar setting in [1] where each attribute has both positive and negative occurrence in a formal context. Membership degrees of positive and negative attributes in intents serve as their lower and upper approximations, respectively, in fuzzy rough set setting.[1] Recently, [19] considered a framework with positive and negative attributes for crisp setting. The present work can be considered to be a generalization of both [1] and [19].

The paper is structured as follows. In Section 2 we recall basic notions important for the rest of the paper; namely residuated lattices, fuzzy sets and fuzzy relations, completely lattice **L**-ordered sets, and formal fuzzy concept analysis. Section 3 introduces the extension of formal concept analysis where a user can select sets ${}^{+}Y$ and ${}^{-}Y$ of positive and negative attributes, respectively. Section 4 then generalizes the results of Section 3 by making ${}^{+}Y$ and ${}^{-}Y$ fuzzy sets. In Section 5 we provide results on attribute implications in the present setting.

2. Preliminaries

*2.1. Residuated lattices, **L**-sets and **L**-relations*

We use complete residuated lattices as basic structures of truth degrees. A complete residuated lattice [2,14,21] is a structure $\mathbf{L} = \langle L, \wedge, \vee, \otimes, \rightarrow, 0, 1\rangle$ such that $\langle L, \wedge, \vee, 0, 1\rangle$ is a complete lattice, i.e. a partially ordered set in which arbitrary infima and suprema exist (the partial order of **L** is denoted by $\leq$); $\langle L, \otimes, 1\rangle$ is a commutative monoid, i.e. $\otimes$ is a binary operation which is commutative, associative, and $a \otimes 1 = a$ for each $a \in L$; $\otimes$ and $\rightarrow$ satisfy adjointness, i.e. $a \otimes b \leq c$ iff $a \leq b \rightarrow c$. Operations $\otimes$ (multiplication) and $\rightarrow$ (residuum) play the role of truth functions of "fuzzy conjunction" and "fuzzy implication." 0 and 1 denote the least and greatest elements. Throughout this work, **L** denotes an arbitrary complete residuated lattice.

Common examples of complete residuated lattices include those defined on the unit interval (i.e. $L = [0, 1]$), $\wedge$ and $\vee$ being minimum and maximum, $\otimes$ being a left-continuous t-norm with the corresponding residuum $\rightarrow$ given by $a \rightarrow b = \max\{c \mid a \otimes c \leq b\}$. The three most important pairs of adjoint operations on the unit interval are

[1] Note that fuzzy interval-valued approaches, e.g. [7,8,10,20], use lower and upper approximations in formal contexts as well but handle both approximations in a positive way.