Information Sciences 317 (2015) 329–348

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

On measurements of covering rough sets based on granules and evidence theory ☆

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ARTICLE INFO

Article history:
Received 20 October 2014
Received in revised form 21 April 2015
Accepted 25 April 2015
Available online 4 May 2015

Keywords:
Covering rough sets
Precision degree
Rough degree
Knowledge granulation
Distinctive degree
Evidence theory

ABSTRACT

Covering rough sets generalize classical rough sets by relaxing partitions of the universe to coverings. The existing research of covering rough sets mainly concentrates on comparing of different kinds of approximation operators and designing algorithms for attribute reductions of decision systems. Less efforts have been made in measurements of covering rough sets. To enrich this field, in this paper we investigate the measurements of covering rough sets based on granules and evidence theory, which can be used to solve more complex practical problems. Firstly, it is shown that the most common sixteen pairs of covering approximation operators can be grouped into five pairs of dual covering approximation operators and six pairs of non-dual covering approximation operators according to definitions based on element, granule, and subsystem. Secondly, we define the rough degree and precision for the purpose of characterizing the uncertainty of these summarized covering approximation operators under a unified framework. The underlying uncertainty of covering rough sets results from the roughness of granulation of coverings. Finally, we investigate the relationships between the covering approximation operators and evidence theory. It shows that some kinds of covering approximation operators can be characterized by belief and plausibility functions, while others do not share this property. The sufficient conditions under which covering approximation operators can be measured by belief and plausibility functions are further explored, and the corresponding counterexamples are given to illustrate those that cannot be measured. Based on above discussions, we develop a basic framework of numerical characterization on measurements of covering rough sets with granules and evidence theory.

1. Introduction

It's common knowledge that the theory of rough sets was originally proposed by Pawlak [25] in 1982. As an effective method to mine knowledge from the digital data with imprecision, vagueness and uncertainty [29,64,75], rough set theory has been applied successfully to many real-world problems in machine learning, pattern recognition, decision analysis, intelligent control, expert systems and so on [26–28,10,11,14,20,32]. So far, research on rough sets has formed many significant fields, such as attribute reduction problems [17,24,35,51], axiomatic systems [52,58,69], generalizations of rough sets

☆ This work is supported by National Natural Science Fund of China (71471060).
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http://dx.doi.org/10.1016/j.ins.2015.04.051
0020-0255/

[42,37,58,69], approximation operator models [33,29,43], mathematical structures of rough sets [68,70,74] and measurements of rough sets [10,11,14,20,21,34,6,49,71,61]. Among these studies, measurement of rough sets is an important and meaningful component. And various measures can be used to numerically characterize the structures of rough sets. Moreover many algorithms of rule acquisition and attribute reduction can be explored correspond to these measures to solve practical problems effectively.

Pawlak's rough sets focus mainly on addressing data sets whose objects can only take a unique discrete value for each attribute. Furthermore, every attribute in these data sets can induce an equivalence relation on the universe of discourses or a partition. However, such an equivalence relation or partition is still restrictive for practical applications because the conditions above cannot always be satisfied in many data sets. To serve this issue, one of effective generalizations of Pawlak's rough sets is to replace equivalence relation with fuzzy relation [12], dominance relation [15], similarity relation [43] or tolerance relation [42]. Another useful generalization of Pawlak's rough sets is to replace the partition by covering of the universe and obtain the covering rough sets [63]. Covering rough set theory can work data sets whose objects take different types of values such as set values, missing values and real values [31,35,36,48,58,57,56,63,3,73]. It can also reveal the internal structure of rough sets. Therefore, covering rough sets are widely used to handle more complex problems as the most meaningful generalization of Pawlak's rough set.

The current research on covering rough sets focus mainly on constructing covering approximation operators and developing algorithms for attribute reductions [5,23,31,35,36,30,44,46,48,54,67,65,68–70]. Within our knowledge, Zakowski first proposed the concept of covering rough set approximations in 1983 [63], where a pair of lower and upper approximation operators are defined by a straightforward generalization of Pawlak's definition. Since then various types of covering lower and upper approximation operators have been proposed [2,33,53,45,65,66,68,22,18,37]. For instance, Bonikowski et al. [2] defined the first type of covering rough sets by using coverings. Pomykala [33] studied the second type of covering rough sets. Wybraniec-Skardowska [53] studied pairs of approximation operators linked together by a different type of relation. Tsang et al. [45] introduced the third type of covering rough sets, where the upper approximation operator was considered to be more reasonable than those of the first and second types. Zhu and Wang [65,68] defined the fourth and the fifth types of covering rough sets from the topological view. Liu and Liao [22] gave the sixth type of covering rough sets. Li [18] introduced two new pairs of covering approximation operators according to define two new coverings of the universe. Qin et al. [37] defined five pairs of covering approximation operators based on neighborhoods. It is well-known that a covering of the universe can induce distinct neighborhood operators, neighborhood systems, coverings, and subsystems of the power set of the universe, which can in turn be used to define different kinds of covering approximation operators [33,7,18,37,50,62,47,65,67,22]. Along with various methods for formation of coverings, the covering lower and upper approximations of a set were defined in various ways, and also a great number of notations are used to denote the covering approximation operators. As a further study on covering rough sets, Samanta and Chakraborty [38] presented sixteen pairs of the most common covering approximation operators and grouped them into two categories in terms of classical set theoretical properties, i.e., ten pairs of dual covering approximation operators and six pairs of non-dual covering approximation operators. However, the classification method of [38] does not reduce the number of covering approximation operators. In other words, [38] merely classified operators in terms of theoretical properties while not analyze the intrinsic structure of the operators themselves. In fact, some of the operators are closely related. For example, the tight pair and loose pair [9] are related. On the other hand, the increasing of amount of definitions and variety of symbolic representations of covering approximation operators also bring great confusion to other subjects in this area, such as measurement of covering rough sets. Therefore, there is a strong need for a method that can group the various types of covering approximation operators into a simpler framework. It was mentioned in [60], by treating an equivalence class as a neighborhood in the element based definition, a partition as a covering in the granule based definition, and a subsystem as a closure system in the subsystem based definition, one can obtain three constructive definitions of covering approximation operators, i.e., an element based definition, a granule based definition and a subsystem based definition. Based on these definitions, covering rough sets can be systematically investigated and categorized. As the result of this method, most kinds of the covering approximation operators can be classified under the framework of Yao and Yao in [60].

Compared with the research on covering approximation operators, less efforts have been made on measurements of covering rough sets [40,16,1,19,72] within a unified framework. Shi and Gong [40] defined rough entropy and the granulation of covering for the purpose of measuring the uncertainty of a covering in the covering approximation space, in which the granulation of covering represents the ability of classifications. Hu et al. [16] investigated the uncertainty measurement of covering rough sets in a Pawlak's approximation space. They were the first to convert a covering approximation space into a Pawlak's approximation space and then create the concepts of information entropy, roughness and rough entropy of a covering. Bianucci and Cattaneo [1] investigated entropy and co-entropy of a covering for incomplete information systems. Li and Yin [19] discussed the knowledge reduction of covering decision systems by defining information entropy and information conditional entropy. Zhu and Wen [72] defined a novel entropy and co-entropy associated to coverings which exhibit the expected monotonicity. Meanwhile, it can also be explored to measure the fineness of any types of covering approximation operators. On the other hand, some authors investigated the relationships between covering rough set theory and evidence theory [13,39,9,4]. For instance, Couso and Dubois [9] investigated the relationships of the loose pair and tight pair of covering approximations and evidence theory, respectively. It shows that the lower and upper approximations of the loose pair can be measured by belief and plausibility functions in evidence theory respectively, but a tight pair does not always share this property. Chen and Li [4] studied the neighborhood-covering rough sets based on evidence theory. It was